PSYCHOMETRIC SCALING AND PREFERENCE METHODS IN SPATIAL ANALYSIS

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Abstract

Psychometric Scaling and Preference Methods in Spatial Analyses

The paper is devoted to the use of multidimensional scaling methods in spatial analyses. These methods originally developed in mathematical psychology open many possibilities to transform ordinal data into metric data.
The paper gives a survey of several methods developed during tha last decade and pays attention to the differences between proximity analyses and preference analyses.
Then the paper demonstrates that both types of analyses can be used in a wide variety of spatial problems in which the input data are measured in non-metric units.
The applicability of multidimensional scaling methods for regional and urban research is illustrated by means of recent research results in the field of recreation behaviour in the Netherlands.

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1. Introduction [1)]

The analysis of human preferences and priority schemes has had a long history in economics. Especially the measurement of utility has been a central issue in economic thinking since 1870, when Menger [1871], Jevons [1871] and Walras [1874] introduced a new theory of value which focussed more explicitly on consumer decisions. In these analyses the preferences for commodities were regarded as a consequence of (cardinal) utility of these commodities. In the debate about utility measurement Edgeworth [1881] and Pareto [1906] emphasized the ordinal character of utility. The mainstream of traditional welfare economics has taken for granted the impossibility of cardinal measurability of utility and the impossibility of interpersonal utility comparisons (see among others De V. Graaff [1957] and Hennipman [1977]). Clearly, these postulates obviate a well-based operational theory of individual and collective choice behaviour.

During the last decades several new contributions to the analysis of rankings of priorities of different actors have been made (cf. Arrow [1963] and Sen [1970]). Special attention has also been devoted in the past to paired comparisons of preferences (cf. Kendall [1948, 1955], Hay [1958] and Buel [1960]).

Against the background of the dilemma 'ordinality - cardinality' it is worth while to pay attention to a set of theories recently developed in the field of psychology. These theories aim to overcome both the ordinality problem of individual choice behaviour and the collective choice problem of multiple actors. This may also lead to a better integration of 'bandwagon' effects, 'Veblen' effects and other external effects which are normally rather hard to incorporate in preference and demand analyses (see also Leibenstein [1976]).

During the last decade there has been an increasing interest in so-called conjoint measurement (see among others Green and Wind [1973], Jungermann and De Zeeuw [1977] and Luce and Tukey [1964]) and in so-called multidimensional scaling techniques. The latter set of methods aims to convert, in general, non-metric (mainly ordinal) information on priorities and preference structures into metric (mainly cardinal) information (see among others Bechtel [1976], Bertier and Bouroche [1970], Coombs [1964], Guttman [1968], Kruskal [1964a, 1964b], Lingoes and Roskam [1971], McGee [1968], Roskam [1975], Shepard [1962], Shepard et al [1972], Torgerson [1958], and Young and Torgerson [1967]. The main part of contributions in this field were made by mathematical psychologists, although

1) The authors are indebted to Piet Rietveld and Henk Voogd for their useful comments on an earlier draft of this paper.

more recently scholars from other disciplines have also shown much interest in these new techniques, like geographers, (cf. Golledge and Rushton [1972] and Tobler et al. [1970], economists (cf. Adelman and Morris [1974]), marketing analysts (cf. Green and Carmone [1970], Green and Rao [1972], Van Raaij [1972], and Shocker and Srinivasan [1974]) and planners (cf. Voogd [1978]).

The aim of multidimensional scaling (MDS) methods is to identify the co-ordinates of N points associated with N objects (commodities or plans, e.g.) such that the interpoint distances demonstrate a maximum correspondence with respect to observed dissimilaraties in perceptions or preferences regarding these N objects. These dissimilarities reflect (subjective) judgments on differences between objects. In other words, on the basis of given dissimilarities the aim is to find a configuration of points such that their distances fit them best. Such a best fit can be achieved by adopting a monotone relationship between dissimilarities and interpoint distances, such that the residual variance of a monotone regression procedure of these distances upon the dissimilarities (the so-called 'stress') is at a minimum.

During the last 5 to 10 years a wide variety of MDS techniques has been developed. In the present paper a brief survey of some essential elements of MDS procedures will be given, followed by a brief discussion of some major classes of MDS methods. The usefulness of these methods for studying spatial behaviour and its underlying preference structure will be illustrated by means of some empirical applications in the field of recreation research.

2. An Introduction to MDS.

The original rationale behind the use of MDS methods was the aim to transform ordinal data, that describe in an N x N paired comparison table the (dis)similarity between N objects, into cardinal units. Assuming a symmetric paired comparison table and omitting the self-dissimilarities on the main diagonal, one has in fact $\frac{1}{2}N(N-1)$ ordinal dissimilarity relationships. The only way to represent these N objects as (cardinal) co-ordinates in a geometric space, is to reduce the number of dimensions. Suppose that the geometric space concerned is the K-dimensional Euclidean space ($K < N$).[1] Then the co-ordinates of the N objects in a K-dimensional space can be gauged due to the fact that the transition from higher to lower dimensions

1) A more precise demarcation of K satisfies the condition: $K \leq \frac{\frac{1}{2}N(N-1)}{N} = \frac{1}{2}(N-1)$.

implies in general many degrees of freedom which can be used to extract cardinal information from the underlying ordinal data structure. The main criterion for assessing the co-ordinates of the N objects in the new K-dimensional space is that these N points have to show a configuration such that the interpoint distances bear a maximum correspondence to the rankings in the initial dissimilarity data.

For example, assume that an individual has to express his priorities for a set of commodities and that he is able to provide a complete ordinal ranking of the commodities concerned which corresponds to his preferences. Each commodity may now be assumed to possess a set of attributes, so that in fact the relative presence of each attribute of the commodities has led implicitly the 'decision-maker' to scale these objects in some way. Assuming K attributes, any given object can then be regarded as existing in a K-dimensional geometric space. The as yet unknown quantity of each attribute perceived by the individual and belonging to a certain commodity can then be related to the corresponding geometric co-ordinate.

It should be noted that this more psychologically oriented approach to consumer demand analysis bears a resemblance to the multi-attribute utility theory proposed among others by Lancaster [1971], although in the latter theory the various relevant attributes are supposed to be known a priori.

If the ordinal dissimilarities are denoted by $\delta_{nn'}$ $(n>n')$, the original paired comparison table Δ for dissimilarities between items is:

$$(2.1.) \quad \Delta = \begin{bmatrix} \ddots & & & \\ \delta_{21} & \ddots & & \\ \vdots & & \ddots & \\ \delta_{N1} & \cdots & \cdots & \delta_{N,N-1} \end{bmatrix}$$

This symmetric matrix is supposed to have a complete ordinal ranking without ties, so that the highest rank number is $\frac{1}{2}$ N(N-1) and the lowest 1.[1] Suppose now that the N objects are to be represented in a K-dimensional space. Then one has to construct the following NxK configuration table which represents the co-ordinates of the N points in a geometric space:

1) It should be noted that the assumption of the absence of ties is by no means necessary and that it can easily be relaxed ; see later. In such a table of perceived dissimilarities the transitivity conditions are not necessarily satisfied.

$$(2.2.) \qquad C = \begin{array}{c|ccc|} & 1 & \cdots & K \\ \hline 1 & x_{11} & \cdots & x_{1K} \\ \vdots & \vdots & & \vdots \\ N & x_{N1} & \cdots & x_{NK} \\ \hline \end{array}$$

Next one may define a distance measure (a Euclidean distance metric, e.g.) between all N points of table (2.2.) (see also Paelinck and Nijkamp [1976]):

$$(2.3. \qquad d_{nn'} = \sqrt{\sum_{k=1}^{K} (x_{nk} - x_{n'k})^2}$$

The best way to achieve an optimal fit between the ordinal data from (2.1.) and the cardinal data from (2.2.) is to impose the condition that the geometric configuration of (2.2.) should be such that the distances represented in (2.3.) do not violate the dissimilarity conditions from (2.1.). This best fit can be achieved by means of a least-squares procedure, viz. by minimizing the (normalized) residual variance ('stress'). This stress-function (or loss function) may have the following shape (although a more general Minkowski metric is also allowed):

$$(2.4.) \qquad S = \sqrt{\frac{\sum_{n,n'} (d_{nn'} - \hat{d}_{nn'})^2}{\sum_{n,n'} d^2_{nn'}}} \quad , \qquad n \neq n'$$

where $d_{nn'}$ is already defined in (2.3.) and where $\hat{d}_{nn'}$ are unknown values (so-called disparities) which should be determined subject to the condition that $\hat{d}_{nn'}$ is in agreement with $\delta_{nn'}$; in other words, $\hat{d}_{nn'} \leq \hat{d}_{nn''}$, whenever $\delta_{nn'} < \delta_{nn''}$. One possible way to determine $\hat{d}_{nn'}$ may be a monotone regression (Kruskal [1964b]), which can be formalized as[2]:

1) Such a stress-function may be regarded as a measure for the degree at which the information from C contradicts that from Δ.

2) See for an alternative procedure among others Colledge and Rushton [1972] and Guttman [1968], who proposed a rank-image procedure.

$$(2.5.)\quad \begin{cases} \min\limits_{\hat{d}_{nn'}} \omega = \sum\limits_{n,n'} \left| d_{nn'} - \hat{d}_{nn'} \right| \\ \text{subject to:} \\ \delta_{nn'} > \delta_{nn''} \longrightarrow \hat{d}_{nn'} > \hat{d}_{nn''} \end{cases}$$

Instead of linear distance functions, any other non-linear distance metric may be used as well. Before (2.5.)can be applied, a first'guess' of $d_{nn'}$,has to be made. This first guess can be made after the determination of an initial configuration of (2.2.); this configuration is often the result of a principal component analysis with K components applied to (2.1.). Given the initial configuration, the initial distances $d_{nn'}$ can be calculated and substituted into (2.5.). Next the monotone regression may be carried out in order to assess an initial value for $\hat{d}_{nn'}$, so that the disparities are in accordance with the (dis)similarities. Thus, $\hat{d}_{nn'}$ is not a specific distance, but a number that is as close as possible to the original distance $d_{nn'}$ while being in accordance with the (dis)similarities.

When the initial values of $\hat{d}_{nn'}$ are substituted into (2.4.), a minimum stress can be calculated (in terms of x_{nk}) by means of a numerical solution procedure for minimizing (2.4.) (for example, by means of a gradient method). The resulting values of the configuration can again be used to assess a new value of $\hat{d}_{nn'}$ etc., until after a number of runs the whole procedure converges.

Instead of the stress function s, other scholars such as Guttman [1968] prefer to use a coefficient of alienation. This coefficient and the procedure involved bear, however, a great resemblance to the stress approach.

So the MDS procedures are based on a whole series of successive steps: (1) the construction of a paired comparison table of dissimilarities $\delta_{nn'}$; (2) the calculation of an initial configuration which is successively manipulated in order to obtain a monotone relationship between the original dissimilarities and the ultimate distances $d_{nn'}$; (3) the use of a set of intermediate variables $\hat{d}_{nn'}$ (so-called disparities) which are determined in accordance with the (dis)similarities and which are used in a stress function (a loss function) so as to minimize the discrepancies between the unknown distances and the disparities; (4) the use of an iterative algorithm which guarantees ultimately a convergence. The whole procedure is represented in a simplified manner in Fig. 1.

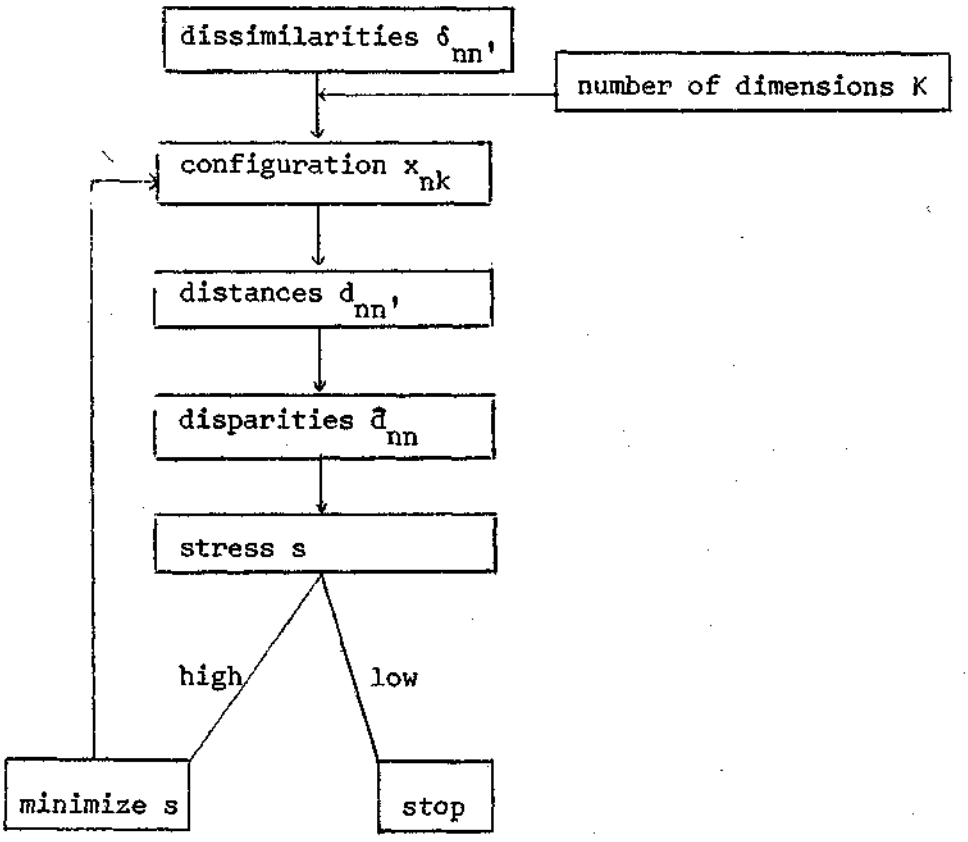

Fig. 1. Simplified representation of MDS procedures.

The conclusions which can be drawn from the ultimate value of the stress function are slightly subjective. Kruskal [1964a] gives the following rules of thumb regarding function (2.4.):

S	Goodness of fit
20%	poor
10%	fair
5%	good
$2\frac{1}{2}$%	excellent
0%	perfect

Another subjective element concerns the choice of the dimension K. Clearly, one should strive at a minimum stress with a minimum number of dimensions involved. Kruskal suggests some aids in this respect: (1) the number of dimensions should be as small as possible, in the sense that an increase in dimension does not lead to a significant decrease in s; (2) the interpretability of the results should not be worsened by increasing the number of dimensions; (3) more dimensions may be taken into account, as the statistical errors of the data are smaller.

It is clear that the ultimate interpretation of the configuration

is also a matter of personal inventiveness of the researcher, so that here again subjective elements may enter. Like in factor analysis, some the results are invariant against a translation and rotation of the axes (provided a Euclidean distance metric is used)[1].

Finally, it should be noted that instead of Euclidean distances any other appropriate distance metric may be used (see for a survey of general Minkowski metrics also Van Delft and Nijkamp [1977]). A discussion of problems of missing data, ties and non-symmetries can also be found in Kruskal [1964a]. An analysis of the problems inherent in the use of a non-symmetric rectangular dissimilarity matrix ('conditional data') can be found in Roskam [1975] (see later on in section 3).

The foregoing presentation of MDS methods hardly made a distinction between various types of MDS procedures more recently developed for a wide variety of data analyses. In the next section some of these methods will be discussed, because they will be applied in the recreation analyses at the end of this paper.

3. Some MDS Methods

3.1. Introduction

The increased interest in MDS techniques has led to a whole set of related methods which serve to analyse similarities (and dissimilarities) between objects or differences in individual and group priorities. Dependent on the problem at hand, a certain structured data input as well as a specific variation of the original MDS scheme has to be used. For example, Coombs [1964] makes a distinction between 4 different types of data on human choices or preferences: (a) preferential choice, (b) single stimulus, (c) stimulus comparison, and (d) similarities data.

Another distinction may be into simple space and joint space problems (see Golledge and Rushton [1972]). In general, simple space problems are related to a stimulus space (goods, e.g.), on the basis of which only metric discrepancies between attributes of objects have to be assessed. Thus, simple space analysis is mainly a proximity analysis which is very useful in the field of perception studies. Some examples of simple space problems are:

- the image of competitive products from the side of a consumer

1) Apart from the non-metric character of MDS-procedures, another difference with respect to factor analysis is the non-linear character of MDS-procedures.

- the perception of the strength of mutually competing firms
- the perceived differences in levels of regional well-being from the side of a policy-maker
- the image of alternative locations for a new firm
- the perception of the qualities of natural areas by a recreant.

Joint space problems are generally slightly more complicated because, in addition to the attributes of an object, also the differences in the priorities or evaluations with regard to the objects themselves among different subjects are depicted. Sometimes the configuration of both objects and subjects is represented in the same metric space, so that it is then possible to draw inferences about the discrepancies between objects, the discrepancies between subjects and the degree of preference of a certain subject with regard to the objects. Therefore, these types of analyses are mostly called preference analyses. Several of these joint space analyses do not only require a square symmetric matrix of dissimilarities as input data, but also a rectangular matrix of (conditional) preference rankings for each subject and/or for each object. Some examples of joint space problems are:

- the identification of qualities of goods in the same space for goods and consumers in order to satisfy the consumer's needs at a maximum degree.
- the analysis of the impact of a change in the attributes of a good on the consumer's perception.
- the analysis of the properties of a shopping centre which attracts spatially dispersed consumers.
- the identification of the main factors determining the perceived attractiveness of recreation areas.

Some of the MDS methods will now be discussed in par. 3.2. - 3.5.

3.2. Proximity Analysis

The most well-known proximity analyses were developed by Kruskal [1964a][1964b], Young and Torgerson [1967], Carroll and Chang [1970], Carroll [1972], and Roskam [1975]. Their computer programs are usually denoted as Mdscal, Torsca, Indscal and Minissa, respectively. The general introduction to MDS techniques from section 2, followed in fact the main lines of Kruskal's proximity analysis. The initial configuration in the Torsca procedure is based on a successive series of principal component analyses. The analysis by Carroll and Chang is mainly an extension of the Torsca procedure and somewhat more advanced, because it in corporates explicitly individual differences in scaled perceptions of objects, so that subjects (consumers,e.g.) weigh the attributes differently.This transition

from a simple space to a joint space analysis leads thus to a common configuration of points in a geometric space, in which the shape of the configuration is effected by the weights implicitly attached by each subject to the various attributes.

In the present paragraph the MDS proximity analysis will be set out somewhat further on the basis of a new variant of the models of Carroll and Chang [1970] and Carroll [1972]. This proximity analysis is based on the assumption that I individuals i (i 1,...,I) have to judge a set of N objects and that there are K attributes underlying perception that are common to all I individuals. Then the following steps are to be undertaken to identify a common configuration of both objects and subjects.

1) Identify a dissimilarity table Δ^i for each individual i. Unlike (2.1.) this table is not necessarily symmetric, so that the elements $\delta^i_{nn'}$ may be related to the dissimilarities in both comparative directions of the objects
2) Fix the dimension of the configuration (say K)
3) Assume for each individual configuration C (see (2.2.)) a set of weights which have the property that

$$(3.1.) \qquad d^i_{nn'} = \sqrt{\sum_k w^i_k (x_{nk} - x_{n'k})^2} \quad ,$$

where w^i_k represents the weight attached by individual i to the (as yet unknown) common attribute k, and where $d^i_{nn'}$ is the distance relationship already defined in section 2. At the beginning of the analysis, all elements of (3.1.), viz. $d^i_{nn'}$, w^i_k and x_{nk} are unknown. An equivalent expression for (3.1.) is:

$$(3.2.) \qquad (d^i_{nn'})^2 = (\underline{x}_n - \underline{x}_{n'})' \hat{w}^i (\underline{x}_n - \underline{x}_{n'}) \quad ,$$

where $\underline{x}_n$ and $\underline{x}_{n'}$ are the nth and n'th row (of order K x1) of C, and where $\hat{w}^i$ is a diagonal matrix with w^i_k (k=1,...,K) as main diagonal elements. The calculation of the elements x_{nk} and the weights w^i_k is based on a series of sub-stages mainly resulting from the procedure developed by Young and Torgerson [1967]:

(3a) Apply (2.4.) and (2.5.) to find for each individual the ultimate configuration, as well as the corresponding disparities, so that the individual results are in agreement with the initial individual preference rankings.

(3b) Construct an auxiliary matrix B^i for each individual i; its

typical elements $b^i_{nn'}$ are calculated as:

(3.3.) $$b^i_{nn'} = (\underline{x}^i_n)' \; (\underline{x}^i_{n'}) \quad ,$$

where $\underline{x}^i_n$ and $\underline{x}^i_{n'}$ are the nth and n'th row of the configuration matrix C^i associated with individual i.

(3c) Next one may assume that $b^i_{nn'}$ can be decomposed into a set of attributes common to all individuals and into a set of weights specific for each individual. In other words:

(3.4.) $$b^i_{nn'} = (\underline{x}^i_n)' \; (\underline{x}^i_{n'})$$
$$= \underline{x}'_n \; \hat{w}^i \; \underline{x}_{n'} \quad , \qquad \forall i$$

in which $b^i_{nn'}$ is known from (3.3.), and $\underline{x}_n$ ($\forall n$) and $\hat{w}^i$ ($\forall i$) are still unknown.

(3d) Use an algorithm to solve both $\underline{x}_n$ and $\hat{w}^i$; one possible way is to use Wold's nonlinear iterative least squares procedure (see Wold [1966]). Given I systems (3.4.), an initial value of all $\underline{x}_n$'s is inserted in (3.4.) ($\forall i$). Next, the elements w^i_k can be estimated by a least-squares procedure. Given these values, one may calibrate the resulting values of $\underline{x}_n$, etc., until a convergent solution has been attained.

Now the results of the whole procedure can be used to calculate the interpoint distances between objects as well as between individuals. Given the ultimate values of $\underline{x}_n$ ($\forall n$), one is now able to represent the configuration C in a K-dimensional group space as a weighted average of the attributes of the objects evaluated by different individuals. The ultimate weights w^i_k can also be depicted in a K-dimensional subject space, so that the weights attached by each individual to the attributes give also rise to a geometric configuration.[1)]

It should be noted that this proximity analysis does not imply any evaluation or choice in favour of a certain object. The proximity analysis is only a cognitive analysis and does not allow to derive inferences about the relative acceptability of the objects. The latter problem is the subject of preference analysis.

An adjusted version of a proximity analysis was constructed by Roskam [1975] in order to combine the advantages of the Kruskal approach with the Guttman approach. This leads to a successive application of monotone regression procedures and rank-image procedures, by making use of both stress values and coefficients of alienation.

1) In this case the axes may not be rotated, although a shrinking or stretching is permitted for the subject space.

3.3. Preference Analysis: External

Preference analyses serve to unfold the individual's preference ordenings of objects into the configuration with various utility models (see Coombs [1964], e.g.). The model described in par. 3.2. was in fact already an intermediate model between a pure proximity analysis and a preference analysis. The preference models include a simultaneous analysis of objects, criteria and individuals, so that the degree of correspondence between judgements regarding products and their properties can be identified. Frequently a distinction is made between an internal and an external analysis. An external analysis is related to the situation where the data to be used are (1) a part of a configuration C of objects (for example, obtained by means of the abovementioned proximity analyses) and (2) a set of rank orderings of the object. The aim of an external analysis is thus to position the individual's preferences a posteriori into the object space.

An internal analysis is a complete preference analysis relating to original data of both objects and individuals, so that the analysis is carried out in a joint space. In this respect one has to assume normally that all individuals perceive the configuration in a similar way, although they may differ in the relative weights to certain attributes or dimensions.

In the present paragraph, only external preference analyses will be discussed, while internal analyses will be discussed in a subsequent paragraph.

There is again a variety of external preference analyses. Examples of these analyses can be found in Carroll [1972], Green and Rao [1972], and Shocker and Srinivasan [1974]. Especially the model developed by Carroll, called Prefmap, is often used in preference analyses and this model will be considered here in a slightly more detailed manner. This preference model aims to identify vectors corresponding to directions of increasing utility for the objects (or of decreasing utility from a certain ideal point onwards). The basic idea of this model is to project individual preference data on an existing configuration of points. The result of such a multidimensional preference analysis is a set of conditional distances between the points of a configuration; these distances are normally measured with respect to an ideal point which is specific for each individual. The underlying assumption is that each individual who has to make a choice among alternatives will compare each alternative with an ideal alternative. He will ultimately prefer the alternative

which is a close as possible to the ideal one (see for a similar approach in multicriteria analysis Van Delft and Nijkamp [1977]). This ideal alternative may be located at an infinite distance (in case of a situation of non-saturation) or somewhere in the vicinity of the existing point configuration (in case of saturation). These two cases give rise to the linear (or vector) model and the concentric model, respectively.

The preference analysis model needs the following information:

(1) a (metric) configuration C (see (2.2.) in which the values of K attributes related to the N objects are known (this information may be the result of a proximity analysis).

(2) a conditional preference table T of order I x N in which I individuals express their priorities regarding N objects as metric or non-metric rankings.[1)]

Then the question is whether certain preference criteria can be identified, whether the relative importance of these criteria can be derived, whether the underlying preference structure (in terms of saturation or non-saturation) can be determined, whether an ideal reference point for each individual can be calculated, and whether the features of each object can be related to the preference criteria. In formal terms one may try to identify for each individual i a point (or vector) $\underline{v}^i$ of order K x 1, such that the rank order of the distances between $\underline{v}^i$ and all $\underline{x}_n'$ s from matrix C are as close as possible to the rank orders of the preferences of individual i expressed in matrix T. The vector $\underline{v}^i$ is usually called the ideal vector. Then the following steps are to be undertaken.[2)]

(1) Define the distance d^2_{in} between the (unknown) ideal vector $\underline{v}^i$ and the (known) vector $\underline{x}_n$ as:

$$(3.5.) \qquad d^2_{in} = (\underline{v}^i - \underline{x}_n)' \, (\underline{v}^i - \underline{x}_n) \qquad \forall n$$

It is clear that (3.5.) allows to derive a set of iso-preference curves related to the various objects.

1) This conditional information comprises only an evaluation for commodities from high to low preferences and does not take into account dissimilarities (see (2.1.)).

2) For the moment the assumption is made that T contains metric information.

(2) Assume the following linear relationship between each element t_{in} $(i=1,\ldots,I;\ n=1,\ldots,N)$ from T and d^2_{in} :

(3.6.) $$t_{in} = \alpha_i d^2_{in} + \varepsilon_{in} \quad ,$$

where ε_{in} is a certain error term to be minimized in a least squares sense, if $\alpha_i > 0$, the distance term and the preference score are positively related to each other.

(3) Assume that the iso-preference curves of individual i are concentric ellipsoides around the ideal point $\underline{v}^i$, while the principal axes may have any direction, except that they should have the same direction for any individual i. This implies that the principal axes are obtained after a orthogonal rotation of the axes of the configuration C, leading to a new configuration C* which is related to the initial configuration by means of an orthogonal transformation matrix P^i of order K x K (i.e., $P^i(P^i)' = I$). Therefore, the typical rows of C* , denoted by $(\underline{x}^*_n)'$, can be re-written as:

(3.7.) $$\underline{x}^*_n = P^i\ \underline{x}_n$$

In the same way the co-ordinates of the ideal point $\underline{v}^i$ may be rewritten as:

(3.8.) $$\underline{v}^{*i} = P^i\ \underline{v}^i$$

(4) Assume a set of weights w^i_k for the attributes of the objects related to each individual i (see (3.1.)). These weights may be unknown. It is clear that the distances are no longer invariant against a rotation.

The last two steps imply that the distance relationship (3.5.) has to be re-defined accordingly, so that

(3.9.) $$\begin{aligned} d^2_{in} &= (\underline{v}^{*i} - \underline{x}^*_n)'\ \hat{w}^i\ (\underline{v}^{*i} - \underline{x}^*_n) \\ &= (\underline{v}^i)'\ (P^i)'\ \hat{w}^i\ P^i\ \underline{v}^i - 2\ (\underline{v}^i)'\ (P^i)'\ \hat{w}^i\ P^i\ \underline{x}_n + \\ &\quad \underline{x}'_n\ (P^i)'\ \hat{w}^i\ P^i\ \underline{x}_n \\ &= (\gamma^i) - 2(\underline{v}^i)'\ R^i\ \underline{x}_n + \underline{x}'_n\ R^i\ \underline{x}_n, \end{aligned}$$

where:

(3.10) $\gamma^i = (\underline{v}^i)' \, (P^i)' \, \hat{w}^i \, P^i \, \underline{v}^i$

and

(3.11.) $R^i = (P^i)' \, \hat{w}^i \, P^i$

Clearly, γ^i is a parameter which is not influenced by the place of the configuration points $\underline{x}_n$. It is evident that the general shape of the iso-preference curves implied by (3.9.) is an ellipsoïde with $\underline{v}^{*i}$ as centre.

The unknown elements in (3.9.) are $\underline{v}^i$, $\hat{w}^i$ and P^i. If instead of an orthogonal transformation specifically related to individual i an orthogonal transformation is carried out which is equal for all individuals, then the reference pattern of all individuals is assumed to be equal, so that the axes of the ellipsoides are equal to those of the group and, hence, correspond to the original axes of the configuration. If also the weights $\hat{w}^i$ are assumed to be equal for each individual i, then the iso-preference curves are centroïdes around the ideal point. Finally, if the ideal point is located as a point at an infinite distance, then the so-called linear (or vector) model arises, in which the iso-preferences contours are orthogonal to the straight line through the origin toward the ideal point. Clearly, the last cases are only special cases of the most general preference model described in (3.5.)-(3.9.). Therefore, the way to assess the ideal point and the related orthogonal transformation will now only be described for the general model. Then, in addition to the abovementioned steps (1)-(4), the following further steps have to be carried out:

(5) Substitute (3.9.) into (3.6.), so that:

(3.12.) $$t_{in} = \alpha_i \gamma^i - 2\alpha_i (\underline{v}^i)' \, R^i \, \underline{x}_n + \alpha_i \, \underline{x}_n' \, R^i \, \underline{x}_n + \varepsilon_{in}$$

$$= \beta_i + (\underline{y}^i)' \, \underline{x}_n + \underline{x}_n' \, V^i \, \underline{x}_n + \varepsilon_{in} \quad ,$$

where:

(3.13.) $\beta_i = \alpha_i \gamma^i$,

(3.14.) $(\underline{y}^i)' = -2\alpha_i \, (\underline{v}^i)' \, R^i = -2 \, (\underline{v}^i)' \, V^i$

and

(3.15.) $V^i = \alpha_i R^i$

(6) Use a multiple least-squares procedure for all individuals i to assess the values of the unknown parameters β_i, $\underline{y}^i$ and V^i of equation (3.12.), given a set of observations of t_{in} and $\underline{x}_n$.

(7) By means of (3.14.) one may calculate the ideal point $\underline{v}^i$ as:

$$(3.16.) \quad \underline{v}^i = -\tfrac{1}{2}\{(V^i)'\}^{-1}\,\underline{y}^i \quad ,$$

while a decomposition of R^i according to (3.11.) may give the values of $\hat{w}^i$ and P^i. It should be noted the ideal point $\underline{v}^i$ is a maximum if $\alpha_i > 0$; otherwise, it will be a minimum. If certain attributes have a negative weight, then the ideal point is essentially a saddlepoint.

Finally, some attention should be paid to the problem of non-metric values of t_{in}. Like in the abovementioned proximity analysis, the problem of non-metric values can again be attached by means of a monotome regression. This implies a iterative procedure, in which the ordinal values of t_{in} are replaced by metric values $\hat{t}_{in}$, such that $\hat{t}_{in}$ is a monotinically non-decreasing function of the ordinal values t_{in}; hence, $\hat{t}_{in}$ represents the metric rank order of the preference data which corresponds best to the interpoint distances. The iterative procedure as such bears a close resemblance to the minimization of the stress function (see (2.4.) and (2.5.)) ; see for an overview of the various variants of this approach Carroll [1972]. New applications of external preference analyses can be found among others in Nievergelt [1971], and Pekelman and Sen [1974].

3.4. Preference Analysis: Internal

As set out in par. 3.3., an internal preference analysis involves a joint space analysis, in which only preference data are used. The assumption is usually made that the individuals perceive the same attributed of objects, but that the relative importance attached to the attributes is different among the individuals. The aim of such an internal preference analysis is to identify the point figuration of the objects and the ideal points of the subjects simultaneously.

In this field also several MDS varieties have been developed, among others by Carroll and Chang [1970], Jacquet-Lagréze [1971] and Roskam [1975]; these different but related approaches are usually called Mdpref, Anapref and Minirsa. The first method is based on a pariwise comparison of preferences for N objects by I individuals, so that the data input consists of I tables of order N x N. Each table has only 3 possible elements[1)], viz. 1 (preferred to), 0 (indifferent) and -1 (not preferred

1) It should be noted that this approach can easily be extended with complete ordinal data.

to). The ultimate joint result of this program is a configuration of objects in a K-dimensional geometric space as well as a set of straight lines through the origin (directions of K-dimensional vectors), which reflect the average direction of the preferences of each individual.

The MDS method developed by Jacquet-Lagrèze is somewhat more complicated. This is also a joint space analysis in which the configuration of objects and the preference directions of individuals are depicted simultaneously. In addition, however, this method includes a classification of individuals according to the relative degree of similarity of their preferences.

The MDS method developed by Roskam is also based on an internal preference analysis of individual priorities regarding objects. This method is especially approprioate for unfolding non-metric preference data and will be discussed here in more detail as a series of successive steps.

(1) Construct a rectangular conditional preference table T of order I x N, which reflects the priority rankings of I individuals for N objects.[1] The individual scales are such that $t_{in} \geq t_{in'}$ implies that individual i prefers object n to n'.

(2) Create a distance metric which is based on the assumption that the attractiveness of object n for individual i can be reflected as a distand d_{in} between a subject point $\underline{x}_i$ and an object point $\underline{y}_n$ in a K-dimensional geometric space, i.e.

$$(3.17.) \qquad d_{in} = \sqrt{\sum_{k=1}^{K} (x_{ik} - y_{nk})^2}$$

The points $\underline{x}_i$ ($\forall$i) and $\underline{y}_n$ ($\forall$n) can be included in a configuration matrix X (of order K x I) and Y (of order K x N), respectively. Clearly, this reduction of non-metric to a K-dimensional configuration bears a close resemblance to the proximity analysis discussed in par. 3.2. The calculation of X and Y is again the result of an iterative algorithm starting off from an initial (normalized) configuration for X and Y based on an adjusted principal component analysis (see also section 2). Given these initial configurations, a new set of $\underline{x}_i$'s and $\underline{y}_n$'s may be calculated such that there is a monotone relationship between t_{in} and d_{in}, in other words:

1) Instead of priority rankings, T may also represent the degree of perceived (dis)similarity between N objects, in which case a transition arises toward a proximity analysis.

(3.18.) $t_{in} \geq t_{in'} \rightarrow d_{in} \geq d_{in'}$

(3) Create again an auxiliary variable $\hat{d}_{in}$ which links the non-metric data t_{in} to the metric data d_{in}. This auxiliary variable can be calculated according to Kruskal's monotone regression procedure and/or Guttman's rank-image procedure (see also (2.5.)).

(4) Minimize the following stress functions:

$$(3.19.) \quad s = \sqrt{\frac{1}{I} \sum_{i=1}^{I} \frac{\sum_{n=1}^{I} (d_{in} - \hat{d}_{in})^2}{\sum_{n=1}^{N} (d_{in} - \bar{d}_i)^2}} ,$$

where $\bar{d}_i$ is defined as:

$$(3.20.) \quad \bar{d}_i = \frac{\sum_{n=1}^{N} d_{in}}{N}$$

This stress function can be minimized by means of standard programming techniques such as gradient algorithms (see also section 2).

(5) Repeat the whole procedure until a converging solution is obtained.

This joint space analysis can be regarded as an adjusted proximity analysis for preference data. It incorporates the advantages of the original Kruskal-Guttman approach for non-metric MDS techniques. The results can be interpreted in a rather straightforward manner.

4. Evalutation of MDS Methods

MDS methods are especially appropriate to extract metric inferences from non-metric multidimensional data, based on a series of fitting procedures and permissable transformations of (dis)similarity or preference data into the cardinal metric of the normal measurement model. The MDS approach is based on the inability of human mind to rank preferences in a metric sense or on the inability of data analysts to provide information in a metric form. MDS methods attempt to attack this problem by seeking for procedures which lead to cardinal transformations of data, so that more quantitative statements can be inferred from ordinal data. In this way these methods constitute a more operational contribution to traditional economic theories on utility measure-

ment and choice behaviour. Furthermore these methods allow a more group-oriented approach of decisions processes, especially due to the possibility to incorporate ordinal compensation elements and weighing schemes.

The major part of MDS techniques is based on the assumption that the researcher should not prescribe to the individuals or decisionmaker which criteria of a plan (or which attributes of an object) should be taken into account. Instead, it is the researcher's task to identify these criteria or attributes ex post on the basis of the ultimate configuration of objects. The interpretation of these criteria or attributes may, however, be subjective. The only way to arrive at a more objectifiable and testable analysis of similarity and preference data is to confront the attributes of each object n (as represented by the ultimate configuration) with exogenous data on the characteristics z_{nk} of the attributes k, so that the interpretation of the axes of the ultimate configuration can be facilitated or even tested by means of a least-square procedure. This would lead to an estimation of the following model:

$$(4.1.) \qquad x_{nk} = \alpha_k z_{nk} + \beta_k + \varepsilon_{nk} \qquad ,\forall n$$

Instead of such a correlation analysis, also a double MDS-technique might be carried out. In that case the results of the ultimate configuration and the exogenous information on the characteristics of the attributes can be plotted in the same space in order to investigate whether the axes more or less coincide. This can also be tested by means of a rank correlation coofficient (see also par. 5).

Another problem may be that individuals are not always capable to represent their preferences regarding objects which are not (yet) known to them, so that direct inquiries may lead to a biased picture of individual preference patterns. This problem might be analyzed further by investigating the similarity between a priori preference rankings and ex post revealed preferences (see for a discussion also Pirie [1976] and Rushton [1969]).

For the moment, our conclusion is that MDS methods open a new perspective for decision-making analysis, although much effort will be required to use them as appropriate tools for predicting choice behaviour of individuals. This can only be done by incorporating more and clearly defined behavioural relationships (including social and physical constraints and risk elements). MDS methods may be very useful to identify the background of dissimiliation between objects or the differences in

5. Applications of MDS-techniques to Recreation Analysis

5.1. Introduction

The MDS methods described in the foregoing sections can be applied to a wide variety of discrepancy and preference analyses: the identification of attributes of commodities which give rise to a perceived dissimilarity between these commodities, the selection of determinants of a choice problem on the basis of a set of priority rankings, the determination of the main motives underlying spatial choice behaviour etc. It is clear that the mobility of our present society rests upon a whole set of determining factors (like push and pull effects, or repulsion and attractiveness effects). In general, it is rather difficult to identify in a quantitative sense the underlying determinants of spatial choice behaviour. In this respect, the use of MDS methods may be fruitful to obtain more insight into the motivations and priorities regarding spatial choice behaviour, e.g. in the field of migration, commuting, shopping, tourism and recreation.

To demonstrate the applicability of MDS techniques for spatial phenomena, in this section two empirical illustrations will be presented based on recent recreation research in the Netherlands. The first application presented in section 5.2. concerns a proximity analysis for different recreation areas around Amsterdam, on which basis a configuration of attributes of these areas can be derived (cf. section 3.2.). This recreation analysis is completed by means of an additional preference analysis based on an external MDS approach (cf.section 3.3.).

The second application presented in section 5.3. focusses on recreation visits to one of the Dutch islands in the Waddensea. By means of an internal preference analysis an attempt is made here to identify simultaneously both the point configuration of recreational purposes of the recreants, the position of the ideal points of the recreants themselves and the attributes of this recreation area (cf. also section 3.4.).

In both approaches the dimension of the configuration space and the subject space is assumed to be 2 (i.e., K=2), in order to facilitate a visual representation and interpretation of the results.

5.2. A Combined Proximity - External Preference Analysis for Reacreation Areas[1)]

The Western part of the Netherlands is densely populated. Therefore, it may be worth while to identify the needs for various types of recreation areas (woods, beaches, lakes etc.) In our analysis 8 different recreation areas are distinguished[2)]. The results presented here are based on a sample of 25 interviewees mainly located in the vicinity of Amsterdam.

The first step of the analysis is the construction of a dissimilarity matrix for the proximity analysis:

$$(5.1.) \qquad = \begin{array}{c} \\ 1 \\ \vdots \\ 8 \end{array} \begin{array}{c} 1 \text{----------} 8 \\ \left[\quad [\delta_{nn'}] \quad \right] \end{array} ,$$

where the elements $\delta_{nn'}$ are natural numbers varying between 1 (perceived strong dissimilarity between a pair of recreation areas) and 4 (perceived strong similarity between a pair of recreation areas). This matrix (at least its right upper part) had to be filled in by each interviewee. On the basis of this information the proximity analysis described in section 3.2. has been applied. The results for the 8 recreation areas are plotted as a two-dimensional configuration in Fig. 1 and 2. Fig. 1 represents the two-dimensional point configuration of the 8 recreation areas; the perceived (dis)similarity between these areas (objects) can be derived from Fig. 1, as far as these (dis)similarities are based on some common underlying characteristics (attributes). Fig. 2 represents the (dis)similarities between the 25 interviewees (subjects) as far as these (dis) similarities accrue from different scores on the perceived proximities between the recreation areas.

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1) For the proximity and preference analysis the Edinburgh Version of Caroll and Chang's Indscal and Prefmap program has been applied, respectively.
2) These areas located around Amsterdam are: Amsterdamse Bos (1), Northsea-coast (2), dunes (sandhills) along the Northsea-coast (3), het Gooi (4), Westeinder Plas (5), Vinkeveense Plassen (6), Loosdrechtse Plassen (7), and IJsselmeer (8). These 8 areas were fairly well-known to the interviewees; areas unknown to the interviewees were excluded a priori from a further analysis.

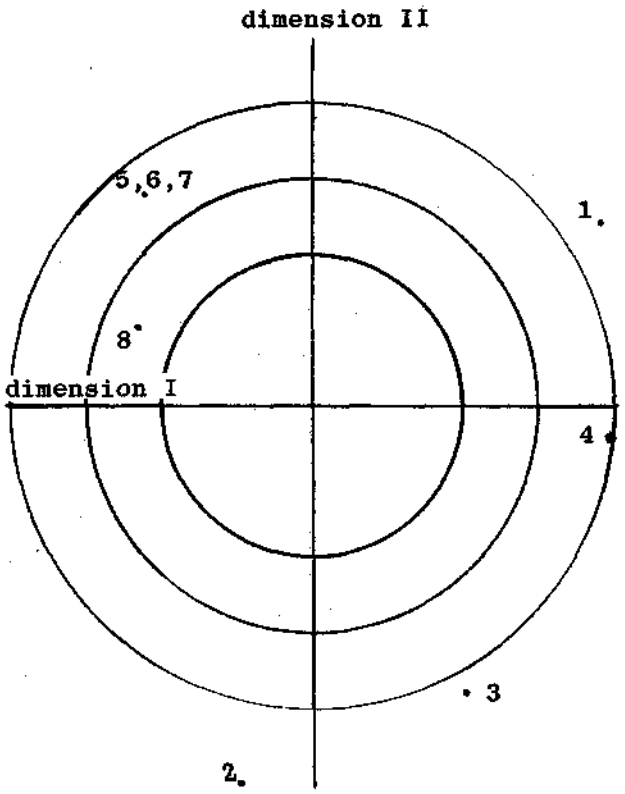

fig. 1 Point (dis)simmilarity configuration of the 8 recreation areas

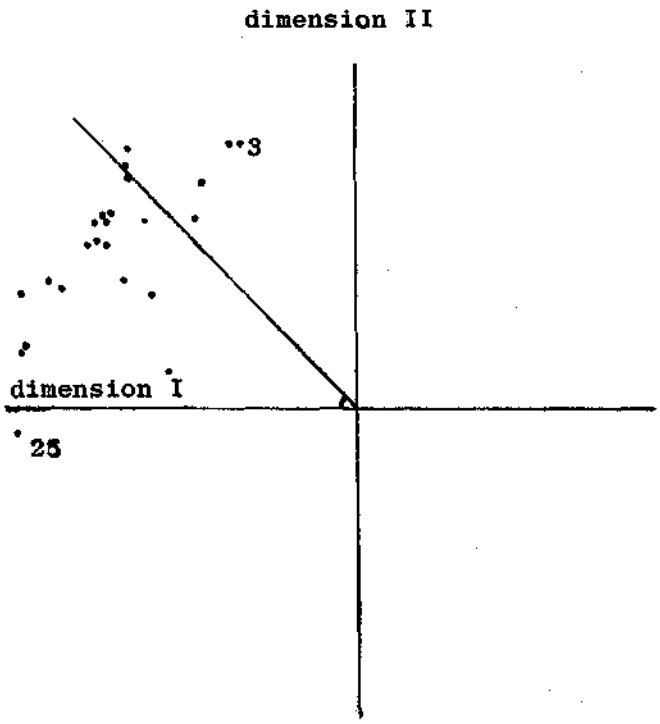

fig. 2 Point (dis)simmilarity configuration of the 25 subjects (subjects on the 45° line give an equal weight to both dimensions)

The pattern of Fig. 1 and 2 appears to be rather stable and reliable, as is reflected by the fairly high value of the correlation coefficients between computed scores and original data for the subjects, which vary between .51 and .96. The following brief comments concerning the results can be made. Fig. 1 demonstrates a distribution of the items over all quadrants. The joint position of the areas (5) - (8) in the left-upper part of Fig. 1 indicates that these areas are perceived more or less equally, which is quite reasonable because these areas are rather popular lakes for sailing, swimming and fishing. The lonely place of area (2) may arise from the specific features of the Northsea coast. The left-hand position of areas (2) and (5) - (8) suggests that the left-hand axis of Fig. 1 can be interpreted in terms of degree of water recreation. This interpretation is supported by the position of areas (1), (3) and (4) which are land recreation areas, so that theright-hand axis reflects the degree of land recreation. Furthermore, areas (1) and (5) - (8) are rather densely congested, man-made artificial areas, whereas areas (2) - (4) are more related to natural areas. Consequently, the upper and lower axis of Fig. 1 may be interpreted in terms of degree of man-made congested areas and natural areas, respectively.

This interpretation may be tested empirically by means of an additional analysis (see later). First, however, the attention will be focussed on the interpretation of Fig. 2 which represents the weights assigned by subjects with

regard to their perceived (dis)similarities between recreation areas. Rather significant discrepancies appear to be present, for example between subject 25 and 3; in the perception of subject 25 the horizontal axis is of major importance, while for subject 3 the vertical axe is of major importance. An intriguing question is whether the perceived dissimilarities between subjects have something to do with differences in their preferences for the characteristics of these areas. This preference analysis of the attributes of the recreation areas will also be dealt with here.

The next step of the analysis was the external preference analysis. The information required to carry out this analysis is a preference ranking of the 8 recreation areas on an arbitrary ordinal scale (ties in the ranking were allowed) by each interviewee. This gives rise to a row vector $\underline{t}'$ of preference scores:

$$(5.2.) \qquad \underline{t}' = \begin{array}{c} 1 - - - - - - - - - - - - - - 8 \\ \left[\qquad\qquad\qquad\qquad\qquad \right] \end{array},$$

where the elements of $\underline{t}'$ are ordinal rankings varying, for instance, from 1 (low priority) to 8 (high priority).

These data combined with the areal configuration from the above-mentioned proximity analysis (see fig. 1) constitute the input for the external preference analysis described in section 3.3. The results for the preference rankings of the interviewees are plotted in the joint configuration-subject space of Fig. 3.

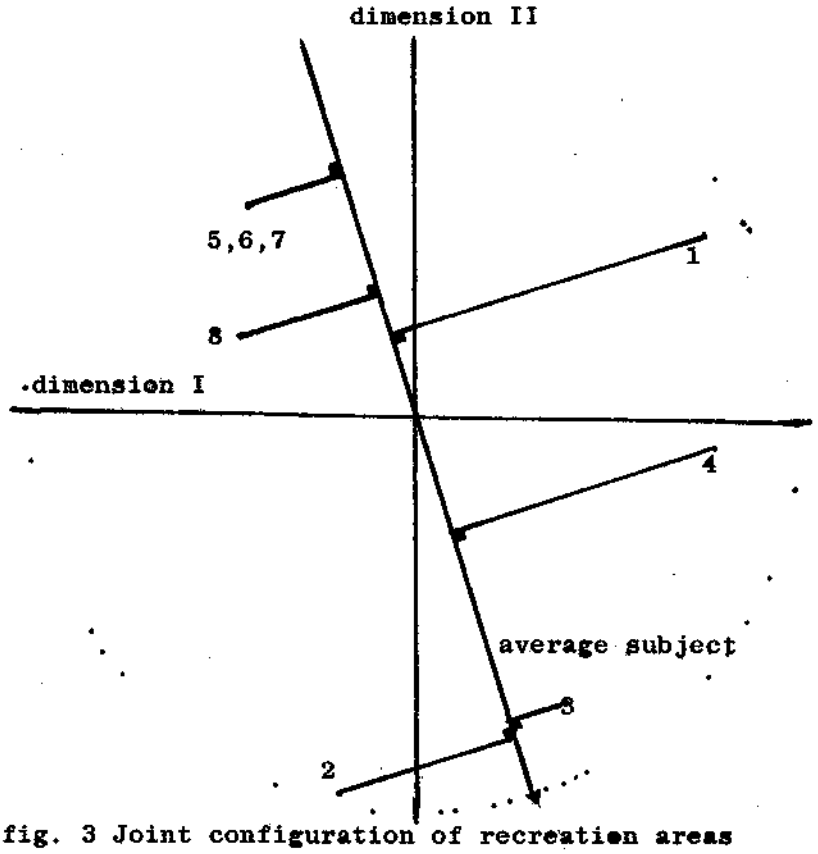

fig. 3 Joint configuration of recreation areas (points), 25 subjects (points= vector directions) and average subject (vector and projection of rank ordering)

Fig. 3 is based on a linear vector model and gives again fairly reliable results: the correlation coefficient for the subjects varies between .70 and 1.00. Clearly, also the general concentric model for iso-preference curves might have been employed (see also Fig. 4), but for the ease of representation and interpretation and due to the fairly good correlation coefficients the use of the vector model is already satisfactory here. The average preference direction of the vector model, given the interpretation of the axes, indicates a rather strong priority for natural areas and to a lesser degree for land recreation areas. It should be noted, however, that the dispersion of the recreation areas as well as of the subjects around this iso-preference line is fairly high, so that not the conclusion can be drawn that a recreation policy should be oriented toward a larger supply or protection of natural areas only. Instead a more refined conclusion may be drawn, viz. that the direction of physical planning and recreational planning should take natural areas and to a lesser degree land areas as a frame of reference for a further extension of recreation areas.

Another conclusion which may be derived from Fig. 3 is that area (2) and (3) are the most favourite recreation areas for the interviewees; this is reflected by the projection of the 8 objects or stimuli (i.e., areas) upon the preference vector of the average subject. The lakes appear to be far less favourable. The latter conclusion corresponds entirely to the foregoing conclusion about the preferences for natural and land areas.

The results of the external preference analysis based on the concentric point model are represented in Fig. 4.

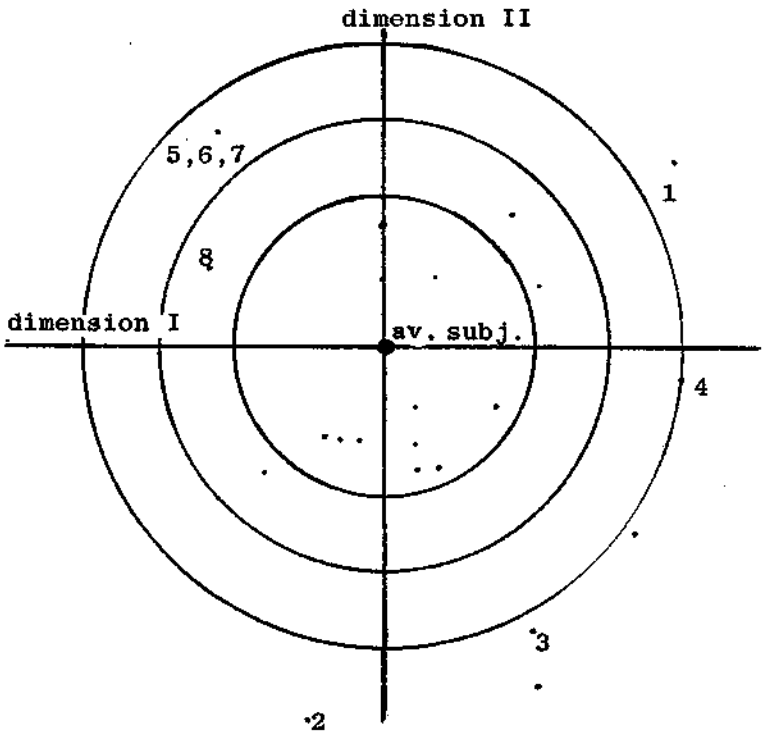

fig.4 Joint configuration of recreation areas (points), 25 subjects (points; some of them are coinciding points) and average subject (point).

The centre of the iso-preference curves is here a negative ideal point, which implies that a further distance from this centre represents a higher preference. Consequently, area (2) has the highest preference and area (8) the lowest one. The rank order of the preferences appears to be almost equal to that of Fig.3, except for an interchange of area (4) and (1). The relative positions and discrepancies of the areas concerned bear also a close resemblance to the picture reflected by Fig. 3.

Finally, the interpretation of the axes may be tested by means of a more quantitatively-oriented correlation analysis (see also (4.1.)).
Such a test requires objectifiable information about some characteristics of the recreation areas (e.g., the percentage part of land or water). For all recreation areas and for all characteristic features this information can be included in a matrix.

A first test may be to relate the quantitative information to the computed configuration by means of a correlation analysis (see (4.1.)). Sometimes, however, the available information on observed characteristics is of an ordinal nature. In that case, the quantitative representation of the exogenous ordinal information on the elements of recreation areas has to be plotted jointly in the same two-dimensional space by means of MDS-techniques. The degree of congruence between the axes of both configurations can be used as a measure for the correctness of the interpretation of the first configuration.

A monotone regression analysis has been carried out for the abovementioned recreation analysis by means of deducing an ordinal scale for the observed characteristics from an interview of several experts. The results appeared to be fairly good (correlation coefficients of .89 for the degree of association between the first characteristic and dimension I, .76 for the degree of association between the second characteristic and dimension II; and moreover, a correlation coefficient for translating the ordinal information to metric information .99), so that there is no need to reject the foregoing interpretation of the axes.

5.3. An Internal Preference Analysis for a Recreation Area[1)]

The Dutch Waddensea is a recreation area of major importance in the Netherlands. Especially during the summer season the islands in the Waddensea suffer from a severe recreation pressure. To identify the behavioural motivations

1) The authors are indebted to Eva Elias, Leon Braat, Pierre Debets and Floris van der Ploeg for their helpful suggestions and/or computational assistence. For this analysis we applied the T.C. version of the University of Amsterdam of Beckam's Minissa program.

of the recreants as well as the economic and environmental repercussions of recreation visits to the islands, a case study for one of the smaller islands has been carried out. One of the main purposes was to identify the characteristic attractiveness elements of the various spatial compartments(such as woods, dunes, beach etc.) of this island. Therefore, an inquiry has been held among the visitors to the island in order to identify the preferences by the recreants. The results presented here are based on a sample from the interviewees.

The data needed for an internal preference analysis are only priority rankings for the recreational items of the island and its spatial compartments. Thus each visitor had to fill in a row vector with ordinal priority rankings for each recreational purpose of the island (see (5.2.) as well as section 3.4.); the total number of recreational purposes distinguished was equal to 6, viz. sports recreation such as trimming, swimming and hiking (1), rest (2), enjoying nature (3), social recreation (cafe's and restaurants etc.) (4), job and study (5), and any other purpose (6).

The results of this preference analysis are represented in Fig. 5 -7. These figures are related to 3 respective categories of recreants, viz. daily recreants, weekend recreants and tourists.

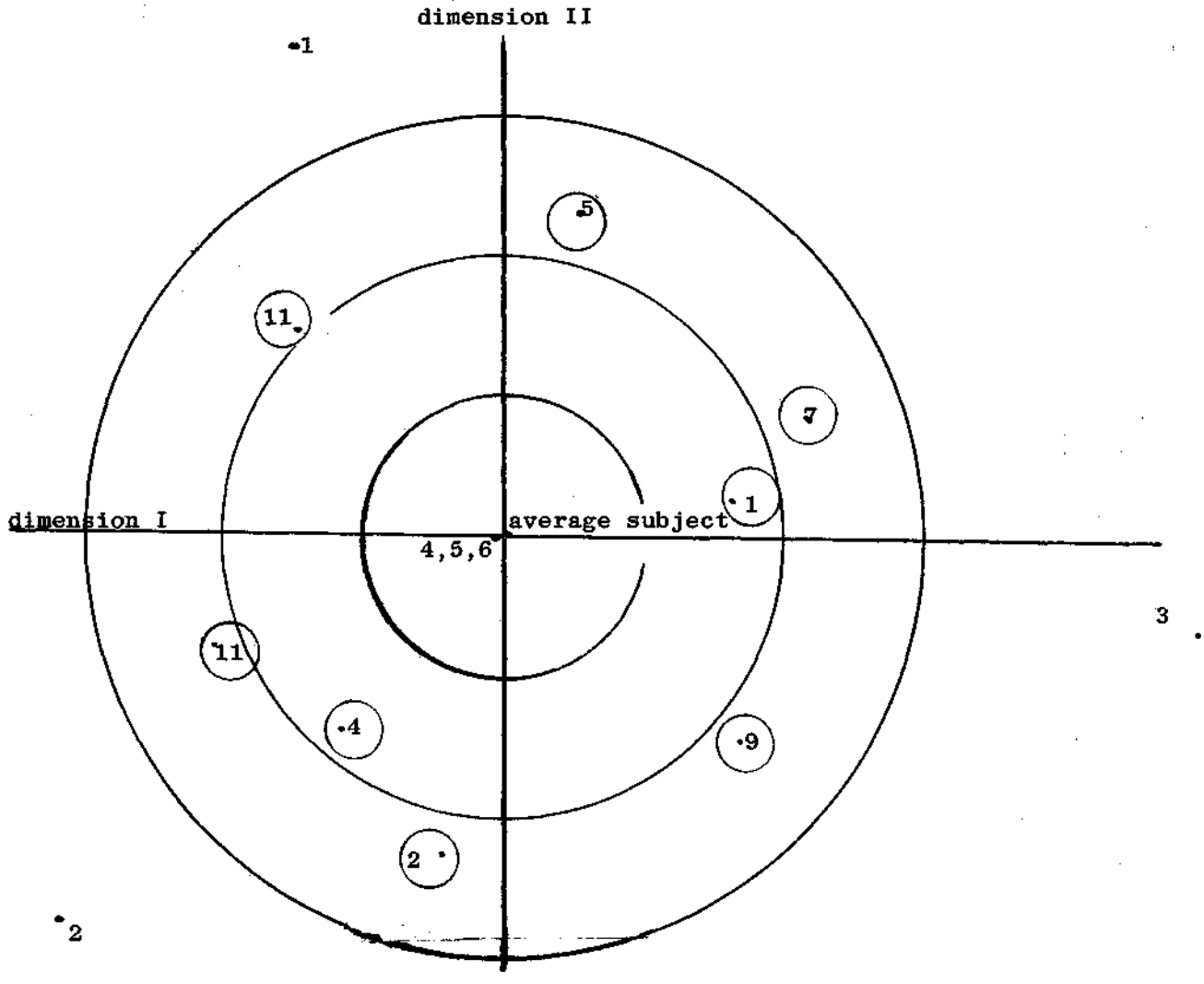

fig.5 Joint space of recreational purposes (1-6) and daily recreants (encircled figures represent the number of recreants on one point). The ideal point of the average subject on the origin is a negative ideal point.

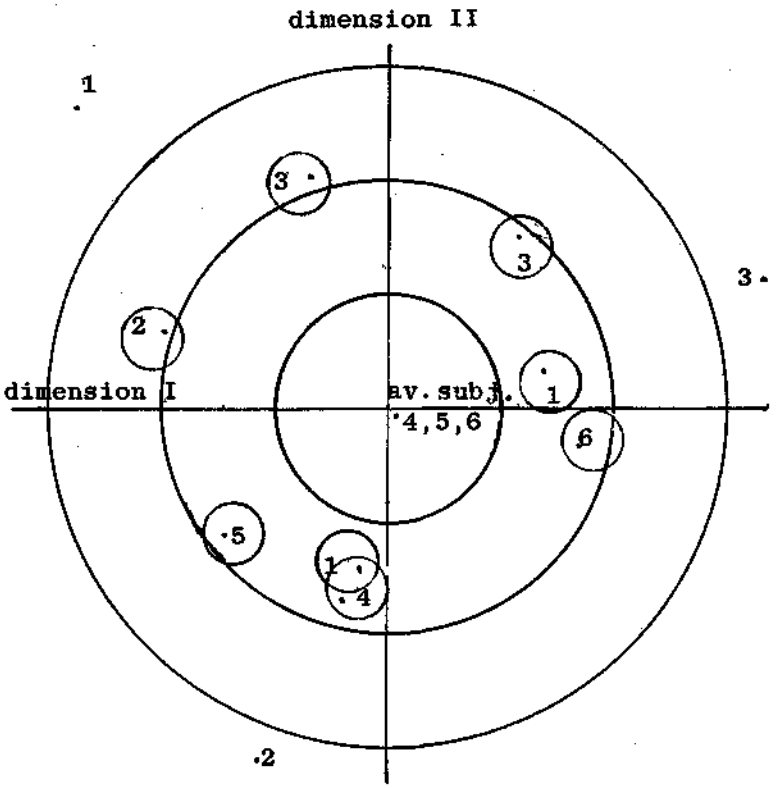

fig.6 Joint space of recreational purposes (1-6) and weekend recreants (encircled figures represent the number of recreants on one point). The ideal point of the average subject on the origin is a negative ideal point.

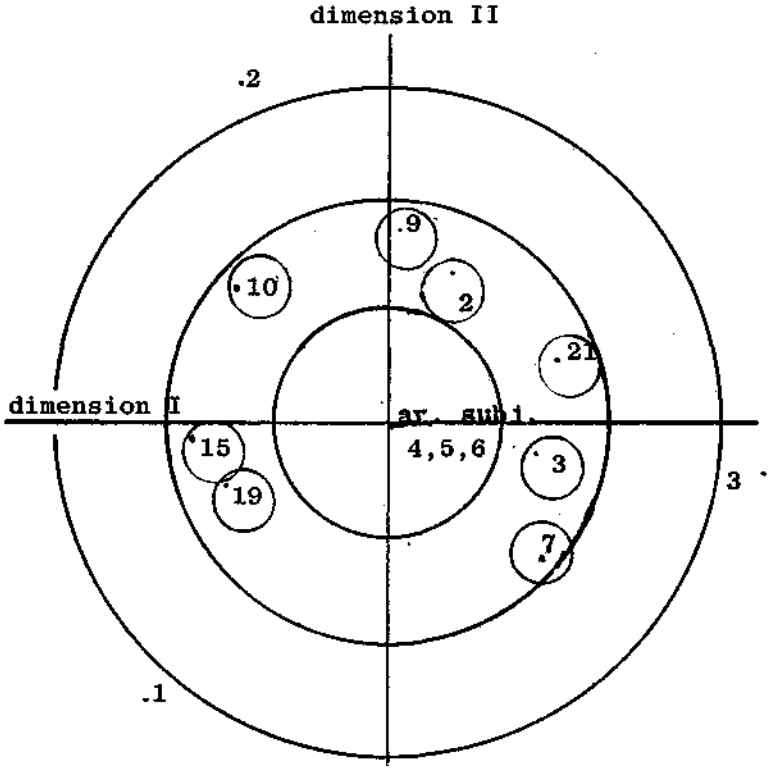

fig.7 Joint space of recreational purposes (1-6) and tourists (encircled figures represent the number of recreants on one point). The ideal point of the average subject on the origin is a negative ideal point.

Fig. 5 represents the joint space of both recreational purposes and daily recreants based on a sample of 50 recreants. The recreational purposes (1) - (3) appear to the most apparent ones, while (4) - (6) receive only minor attention. Given the configuration of the item space, the vertical axis can be interpreted as the degree of activeness (upper part) or the degree of passiveness (lower part) desired by the recreants. In an analogous manner the left-hand axis may be regarded as a measure for the degree of man-made environment, while the right-hand corresponds to the degree of natural environment. Due to the fact that especially items (1) - (3) have received a high priority from the interviewees and items (4) - (6) a very low priority, the first three items may be regarded as the most discriminating items for identifying the axes.

The subject space appears to reflect a fairly diffuse pattern through all quadrants, so that the conclusion can be drawn that these daily recreants have, on the average, a less pronounced priority for one of the attributes of the items; hence, both the degree of natural environment and the degree of active recreation are motives for visiting the island concerned.

Fig. 6 is based on a sample of 25 recreants. It provides a picture which is fairly similar to Fig. 5, except for the position of item (3) which rises now above the horizontal axis. This result indicates that weekend-recreants are inclined to use natural environments in a more active way than daily recreants. The spatial configuration of the subjects shows again a diffuse pattern, although a certain preference for natural environments and active recreation can be inferred.

Fig. 7 is related to tourists spending more than 3 days on the island, based on a sample of 86 interviewees. Here again the vertical axis may be interpreted in terms of degree of activeness of the recreants. Compared with Fig. 5 the third item (nature) is located at the same side as item (1), which suggests that the way of enjoying nature by tourists is more actively-oriented than by daily recreants, although less active then the weekend recreants. These results correspond to those from Fig. 6. The configuration of the tourists in the subject space shows again a diffuse pattern, although two main groups of recreants can be identified, viz. one group that prefers a passive recreation in nature (34 subjects) and one group that prefers an active recreations in a man-made environment (32 subjects).

The analysis presented so far is a part of a larger research project in which the discriminating features of the compartments of the island (the 'supply profile') are confronted with human priorities for recreational purposes (the 'demand profile'). On the basis of this analysis an attempt has to be made to identify especially those areas for which a serious recreational pressure may be expected, given the natural and artificial conditions of these compartments and given the priority rankings of recreants. In this respect the foregoing geometric scaling procedures can be used to detect the spatial behavioural backgrounds of congestion in spatial systems.

6. Conclusion

MDS methods can be regarded as operational tools to identify the components and backgrounds of spatial behaviour. Especially in the field of common goods (such as many recreation areas) it is extremely important to assess human priorities (cf. the free riders problem). In this respect the MDS approach is very fruitful, particularly because this approach attempts to translate 'soft' (ordinal) preference information into 'hard' (cardinal) preference information. On the basis of this approach, a revealed preference analysis is not necessary to extract from price-demand (or in general, market) relationships priority rankings. Instead, after a set of simple interview questions the characteristic features of individual and collective priorities can be detected and visualized by means of a cardinal point figuration. The approach presented in the foregoing sections can also be regarded as a further operationalization of Lancaster's multi-attribute utility theory. The flexibility of MDS methods is also reflected by the diverse variants such as proximity analysis and (internal and external) preference analysis. The possibility to identify an ideal point for an average subject in the joint configuration-subject space and/or a few ideal points for main groups (clusters) of recreants is also important, because this position may provide a frame of reference for public decision-making and planning on which basis the qualities of other areas as well as new development programs can be judged. The interpretation of a configuration appears to be sometimes a less easy task, so that it is in general a good strategy to link the results of a MDS analysis in an objectifiable way (e.g., by means of a least squares approach) to observed quantitative characteristics of the area itself. This procedure can be regarded as a test on the interpretation of the results.

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RESEARCH MEMORANDA:

1977-1	L. Hordijk/P. Nijkamp,	Estimation of Spatiotemporal Models. New directions via distributed lags and Markov schemes.
1977-2	P. Nijkamp,	Gravity and entropy models: The state of the art.
1978-1	J. Klaassen,	Valutaproblemen in de Jaarrekening.