

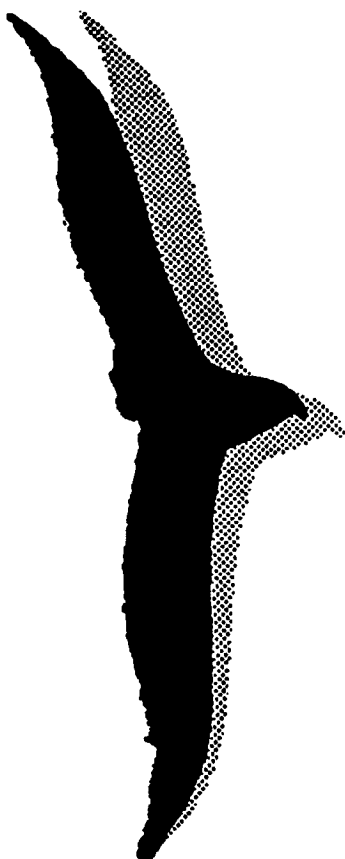
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Jaap H. Abbring

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Commitment and Wage Bargaining in Job Matching Models with Costly Search*

Jaap H. Abbring
Vrije Universiteit Amsterdam†

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Abstract

This paper provides a new strategic underpinning of the axiomatic Nash bargaining solution that is widely applied in search-matching models of the labor market. This ‘intertemporal surplus sharing’ (ISS) solution is usually defended as the unique subgame perfect equilibrium of a strategic bargaining model in which the risk of breakdown grows infinitely large. We argue that such extreme assumptions on the risk of breakdown during disagreement are unattractive in the context of a search-matching model. We then show that ISS arises for arbitrary breakdown rates if agents can precommit to search during disagreement.

*This paper draws on Chapter 3 of my Ph.D. thesis (Abbring, 1997).

†Department of Economics, Vrije Universiteit (Free University), De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Phone: +31-20-444.6047. Fax: +31-20-444.6005. Email: jabbring@econ.vu.nl.
[Http://mail.tinbinst.nl/~abbring](http://mail.tinbinst.nl/~abbring)

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1 Introduction

In job matching markets with costly search quasi-rents arise as soon as agents are matched. Typically, it is impossible to write *ex ante* contracts on the division of such quasi-rents from future matches, and wages are negotiated *ex post*, i.e. after agents have been matched.¹ In the job matching literature, it is usually assumed that a matched worker and firm divide the value of the match according to an axiomatic Nash (1950) bargaining solution. It is well known that the resulting wage depends critically on the exact specification of the Nash bargaining problem, in particular the choice of the so called threat points (Binmore, Rubinstein and Wolinsky, 1986).

The seminal papers on matching and search use the particular Nash bargaining solution in which the threat points correspond to the outside options available to each agent, i.e. to be unmatched and searching for alternative **partners**.² This solution exogenously fixes the share of each agent in the quasi-rent, i.e. the expected search costs (and foregone earnings) incurred by both agents to form a match, and can thus be described as ‘intertemporal surplus sharing’ (ISS). Unfortunately, ISS is by no means the only Nash solution that can be applied. A frequently encountered alternative allocates fixed shares in the ‘instantaneous surplus’ to each agent. This instantaneous surplus is the marginal flow product of labor net of the instantaneous values of leisure to both agents, and is unrelated to the expected search costs. This implies that the corresponding wages, unlike ISS wages, do not vary with aggregate labor market conditions, for given productivity and values of leisure. Thus, we do not only have a variety of *a priori* equally attractive Nash solutions to wage setting, but these solutions also have different theoretical and empirical implications. This explains the efforts that are made in the literature to support axiomatic Nash solutions as equilibria of strategic bargaining **games**.³ In particular, the strategic approach provides insights in the strategic role of outside options and disagreement payoffs, and can therefore guide the specification of the threat points in the Nash solution.

ISS is usually defended as the unique **subgame** perfect equilibrium (SPE) of a **strate-**

¹ *Ex ante* contracts are inconsistent with search frictions (Mortensen, 1982). An alternative to *ex post* bargaining arises if agents on one side of the market are committed to wages that are posted *ex ante*. Such unilateral *ex ante* wage setting would take us from the class of models we consider in this paper to the wage posting models studied by Burdett and Mortensen (1998) and Mortensen (1990). Of related interest is the work on competitive search by Moen (1997) and Shimer (1996), who effectively allow firms to **precommit** to and announce wages and workers to search non-randomly for jobs with specific wages.

²See Diamond and Maskin (1979), Diamond (1982), Mortensen (1982), Pissarides (1984, 1985 and 1990). Note that ISS is also arguably the most popular solution in the more recent literature (e.g., Mortensen and Pissarides, 1999).

³This is known as the ‘Nash program’, and can be traced back to Nash (1953).

gic sequential bargaining game along the lines of Rubinstein (1982) that is fully driven by the risk of breakdown (e.g., Mortensen and Pissarides, 1999). In the context of a search-matching model, this requires the rate of breakdown of bargaining matches to grow infinitely large. This cannot be caused by the arrival of shocks destroying the productive capacity of the match, as this would imply that producing matches break down infinitely fast as well. Then, all gains from trade vanish and the model would be trivial. Thus, it is an extreme, and therefore hard to defend, feature of the bargaining environment itself. Furthermore, for any positive interval between two bargaining rounds, the bargaining game with extreme risk of breakdown is an ultimatum game. Such a game predicts the existence of two extreme wages, each allocating the entire surplus to either one of the agents, with ISS only holding in expectation. This creates strong incentives for search on the job, which is usually assumed away in search-matching models with homogeneous agents.

In this paper we show that ISS can be supported without relying on extreme differences between bargaining and producing matches. In particular, we show that ISS arises in the unique SPE of a bargaining game in which agents can precommit to search during disagreement. This result is derived independently of the risk of breakdown during the bargaining game, and therefore does not suffer from the critique above. Also, it can be contrasted with earlier research on bargaining with endogenous search, which shows that ISS does not arise without commitment.

The paper is organized as follows. In Section 2 we introduce the search and matching model. Section 3 defines the bargaining problem in a partial framework in which market opportunities are taken to be exogenous. Section 4 discusses bargaining solutions with exogenous search during disagreement. We use these results to sketch the arguments usually applied to defend various axiomatic solutions in the job matching literature. Section 5 then argues that the standard ISS solution arises if we allow agents to precommit to search in the disagreement state, which is the main result of the paper. Section 6 concludes.

2 The search and matching model

Our model is a standard homogeneous job matching model with endogenous search intensities along the lines of Pissarides (1990). Point of departure is a continuous time world with two distinct homogeneous masses of infinitely small agents, workers (w) and firms (f). Both workers and firms are risk neutral and maximize payoffs, which are discounted at a rate ρ . Workers are endowed with a constant stream of an indivisible unit of leisure. Leisure is indivisible in the sense that, at each point in time, current leisure can either be fully consummated or fully traded with firms, after which we will call it labor. Firms

are entities that can flexibly employ capital. However, capital is only productive if it is matched to labor. We assume that firms cannot buy more than one unit of labor, or employ more than one worker, simultaneously. A matched firm-worker pair can produce a flow of output π . We will refer to producing agents as ‘employed’, and attach the label ‘unemployed’ to agents that are not producing, firms and workers alike. We will see shortly that the latter category does not only include unmatched agents, but also matched and bargaining agents.

Trade of labor takes place in the labor market and is characterized by a trade friction: matches with agents of the opposite type are only realized after a time-consuming and costly random process of bilateral search. Following the literature, we assume that production is a fully specialized activity: agents can only participate in the matching process if they are unemployed.⁴ All unemployed agents of each type face the same search technology. An agent of type i who chooses a search effort $s_i \geq 0$ meets agents of the other type at a rate s_i , and incurs search costs $c_i(s_i)$, $i = w, f$.⁵ The cost functions are twice differentiable, and satisfy $c_i(0) = 0$, $c_i'(0) = 0$, $c_i'(s) > 0$ for $s > 0$, $c_i'(\infty) = \infty$, and $c_i''(s) > 0$ for $s \geq 0$, $i = w, f$.⁶ Unemployed agents do not only incur search costs, but also receive a flow of ‘leisure’ payoffs ζ_i . Thus, an unemployed agent of type i exerting search effort s has flow payoffs $\zeta_i - c_i(s)$. We assume that $\pi > \zeta_w + \zeta_f$, so that there are wages at which agents of both types strictly prefer trade over unemployment.

Because of the trade friction, each match is to some extent a specific relationship, which generates a quasi-rent that has to be divided among the worker and the firm. We assume that, right after being matched, the agents enter a bargaining game in which they negotiate a flow of compensation payments, or wages, the firm has to pay the worker for providing labor. In particular, compensation is arranged by a contract specifying a constant wage flow from the firm to the worker, and which cannot be renegotiated.⁷ In

⁴See for instance Pissarides (1990). Alternatively one could assume that agents face ‘full bargaining frictions’, in the sense of Shaked and Sutton (1984), and cannot bargain with a new partner and work on the old job at the same time. Together, these two assumptions ensure that workers will not search **on-the-job**, even if it is possible. Abbring (1998) explores the incentives for on-the-job search in a homogeneous matching model with full bargaining frictions, but in which agents can ‘bargain on-the-job’.

⁵In a fully developed matching model, we would specify the contact rates as $s_i q_i$, for some $q_i > 0$, and the q_i would be endogenized by a matching function relating the contact rates to aggregate labor market conditions (see Pissarides, 1990). In this paper we focus on bargaining solutions that are valid irrespective of the way the aggregate matching rate is determined. Thus, we derive results for given q_i . As we have general cost functions, there is no loss of generality by just fixing $q_i = 1$, $i = w, f$.

⁶We could allow for a fixed cost in search, but this would only be relevant in a fully developed matching model, in which participation decisions of agents are modeled.

⁷As producing matches are only affected by one type of shock, which is thought to destroy all gains from trade, this does not lead to any inefficiencies. As such, renegotiation is not a relevant issue in our model. We return to this in the discussion in Section 6.

equilibrium, agreement upon a wage w will be reached immediately after the match is formed, and production will commence, yielding a flow of payoffs w for the worker and payoffs $\pi - w$ for the firm. The exact bargaining procedure will be discussed in Section 3.

Finally, producing matches dissolve at a rate σ , leaving both agents involved unmatched. This exogenous breakup of employment relationships is thought to be caused by shocks that destroy all production potential of the match. Apart from these shocks we will not allow for any other shocks hitting employment.

3 The bargaining problem

The main argument of the paper can be introduced in a partial analysis, in which we focus on a single bargaining problem in a single match, taking the market wage $\bar{w}$, i.e. the expected wage in matches to alternative partners, as given.⁸ We shortly discuss the extension to market equilibrium, in which it is explicitly recognized that market wages should be consistent with decentralized bargaining outcomes, in Section 6. Also note that, even though we do not explicitly discuss steady state conditions for our matching model, we restrict attention to stationary aggregate conditions throughout the paper. In the partial analysis that follows this means that all parameters of the model, including the market wage, are invariant over time.

Thus, suppose we have a single worker and firm that are just matched and have to negotiate a wage. We model this negotiation by a modified random offers version of the alternating offers model by Rubinstein (1982). Suppose that the pair is matched at time 0, and suppose that bargaining proceeds in rounds that cover consecutive time intervals $[\Delta n, \Delta n + \Delta)$, $n = 0, 1, 2, \dots$, for some $A > 0$. Bargaining is then organized according to

Procedure 1. Bargaining starts with round $n = 0$, and each round proceeds in 4 steps.

n.0 At time A_n , nature selects the proposing player: with probability β the worker is selected and with probability $1 - \beta$ the firm, for some $0 < \beta < 1$.

n.1 The selected player proposes a wage contract w .

n.2 The responding player chooses either to accept or to reject w . Upon acceptance bargaining ends and production starts immediately with compensation w . Upon rejection both partners enter the search state *n.3*.

n.3 Both players search with given intensities s_w and s_f for alternative matches, and face an exogenous hazard of breakdown τ , during the interval $(A_n, A_n + A)$. Upon

⁸Note that we can safely restrict attention to expected wages because of risk neutrality and the implied linearity in wages of all value functions.

meeting another match partner, the agent leaves immediately, and trades with the new partner against the market wage $\bar{w}$, leaving the other agent unmatched. If the match breaks down exogenously, both agents end up in the unmatched state. If, during $n.3$, neither agent has contacted an alternative partner, and if the match has not broken down exogenously, they enter $(n + 1) .O$.

Steps $n.0$, $n.1$ and $n.2$ occur sequentially in a period of length 0.

We assume that both agents perfectly recall all past offers and events. This implicitly defines the relevant histories and strategies. Note that, in our partial analysis, we do not allow the agents to leave the bargaining process for the unmatched state voluntarily, i.e. to take up the immediately available outside option. In Section 6 we argue, however, that we can allow for such outside options in a market equilibrium analysis without changing the partial equilibrium results, as outside options cannot be binding in market equilibrium. Also, $n.3$ does not allow an agent who has contacted an alternative partner to choose between the old and the new partner. However, we will argue in Section 6 that leaving for the new partner is optimal in market equilibrium. Finally, note that we allow the exogenous rate of breakdown τ to be different from the rate of breakdown during production σ . This will be useful in explaining the relation with frequently used axiomatic bargaining solutions.

We close our partial model by specifying the payoffs accruing to the agents when the bargaining procedure above breaks down, which are the discounted values V_w^* and V_f^* of the unmatched state to workers and firms searching **optimally**.⁹ These values depend on the leisure payoffs, search costs, and the expected capital gain of contacting a job offering a market wage $\bar{w}$. Let $W_w(w)$ and $W_f(w)$ denote the present values of employment at a wage w . Note that the total value of the match, which we denote by $\bar{W} := W_w(w) + W_f(w)$, is independent of w . The values of the unmatched state $V_i(s)$ to agent i searching with intensity s is implicitly given by the asset equation¹⁰

$$\rho V_i(s) = \zeta_i - c_i(s) + s(W_i(\bar{w}) - V_i(s)), \quad i = w, f.$$

We anticipate that in market equilibrium $\zeta_w < \bar{w} < \pi - \zeta_f$, which ensures an internal solution to the optimal search problems. Because of stationarity, it is easy to solve for optimal search by unmatched agents. The resulting (optimal) values of unemployment,

⁹Note that these are also the payoffs accruing to the agents when their employment relationship is terminated. However, we will present all results in terms of the present values of employment, and only exploit some elements of the structure of these value functions, like monotonicity in wages.

¹⁰As bargains are struck without delay in equilibrium, these value functions can be constructed without distinguishing bargaining as a separate state.

and therefore the breakdown payoffs, are given by the Bellman equation

$$\rho V_i^* = \max_s \{ \zeta_i - c_i(s) + s(W_i(\bar{w}) - V_i^*) \}, \quad (1)$$

and the corresponding first order condition for the optimal unemployed search intensity, say s_i^* , by

$$c_i'(s_i^*) = W_i(\bar{w}) - V_i^*,$$

which simply states that marginal costs and benefits of search should be equal. Note that the convexity of the search costs ensures uniqueness of this solution.

4 Nash surplus sharing and exogenous search efforts

A partial equilibrium of our search-matching and bargaining model is a SPE of the bargaining model, given optimal search s_i^* in unemployment. We first characterize SPE for given, i.e. exogenous, search efforts s_i in the disagreement state. It is well known that the bargaining game following Procedure 1 has a unique SPE in which a bargain is struck without delay. Furthermore, for $A > 0$, the resulting wage w in this SPE depends on the identity of the agent that is drawn by nature to make the first proposal. Let w^i denote the SPE wage if agent i is proposing first. Then, the expected division of $\bar{W}$ in SPE can be represented by the expected wage, $Ew = \beta w^w + (1 - \beta)w^f$.

Before characterizing the SPE, it is convenient to introduce the present value of disagreement D_i to agent i , which is implicitly given by

$$\rho D_i(s_i, s_{j(i)}) = (\tau + s_{j(i)})(V_i^* - D_i(s_i, s_{j(i)})) + \zeta_i - c_i(s_i) + s_i(W_i(\bar{w}) - D_i(s_i, s_{j(i)})),$$

for $i = w, f$. Here, $j : \{w, f\} \rightarrow \{w, f\}$ adds the opponent to each player, i.e. $j(w) = f$ and $j(f) = w$. The flow value ρD_i of perpetual disagreement is the sum of the expected capital gain because of breakdown, the utility of leisure net of search costs, and the expected capital gain attached to the flow of market opportunities. Consistent with earlier notation, we write $\bar{D}(s_w, s_f) := D_w(s_w, s_f) + D_f(s_f, s_w)$. Note that D_i is defined for given search efforts s_i and $s_{j(i)}$, which are not necessarily equal to the optimal efforts of the unemployed, s_i^* and $s_{j(i)}^*$. For future reference, it is also useful to note that $D_i(s_i, s_{j(i)}) \rightarrow V_i^*$ for $\tau \rightarrow \infty$, for all s_i and $s_{j(i)}$. Furthermore, $D_i(s_i, s_{j(i)})$ attains a unique maximum of V_i^* at $s_i = s_i^*$, for all $s_{j(i)}$ and parameter values. Thus, s_i^* maximizes the value of unemployment to agent i , irrespective of whether agent i is unmatched or bargaining. In turn, this implies that $\bar{D}(s_w, s_f) < \bar{W}$.

The next proposition characterizes the SPE in terms of Ew and $w^w - w^f$.

Proposition 1. *The bargaining game following Procedure 1 has a unique SPE in which a bargain is struck without delay, and in which $\mathbf{E}w$ satisfies*

$$W_w(\mathbf{E}w) = D_w(s_w, s_f) + \beta(\bar{W} - \bar{D}(s_w, s_f)). \quad (2)$$

Furthermore, $w^w \leftarrow wf \rightarrow 0$ for $A \downarrow 0$.

Proof. Similar to Rubinstein and Wolinsky (1985). For completeness, a sketch of the proof for our version of the model is provided in the Appendix. $\square$

In general, equation (2) can be interpreted as an axiomatic Nash (1950) bargaining solution to the division of a (gross) surplus $\bar{W}$ in which the present values of disagreement assume the role of the threat points. This general form reduces to more well known Nash solutions in special cases.

ISS arises if $D_i = V_i^*$, i.e. if $s_i = s_i^*$ or $\tau \rightarrow \infty$. In this solution, agents are allocated fixed shares in the quasi-rent. This quasi-rent corresponds to the expected search costs (including foregone earnings) incurred by both agents, and is the wedge between the current match and the outside options. The fact that wages depend on the costs of forming alternative matches motivates the use of the adjective ‘intertemporal’ in ISS. Alternatively, the agents in a match could split the (gross) instantaneous surplus π according to an axiomatic Nash bargaining solution in which the threat points are given by ζ_w and ζ_f . Obviously, both bargaining outcomes have different theoretical and empirical implications, which explains the efforts that are made in the literature to support the Nash solutions as equilibria of attractive strategic bargaining games.

ISS is the solution to the wage bargaining problem proposed by Diamond and Maskin (1979), Diamond (1982), Mortensen (1982) and Pissarides (1984, 1985), and is frequently applied in the subsequent search and matching literature (see, for examples, Pissarides, 1990, and Mortensen and Pissarides, 1999). ISS is usually defended as the SPE of a strategic bargaining game that is driven by the risk of breakdown. In terms of our model, this line of defense rests on the fact that the expected wage in SPE converges to the wage that splits the intertemporal surplus if $\tau \rightarrow \infty$, given the other parameters. The limiting game that arises for fixed A is an ultimatum game, in which bargaining breaks down almost surely after rejection of the first offer. This is the strategic underpinning of ISS suggested by Mortensen and Pissarides (1999). In SPE, the player i selected by nature to propose in the first bargaining round will propose a wage w that provides a value of employment $W_{j(i)}(w)$ to the other player $j(i)$ just equal to his or her breakdown payoff $V_{j(i)}^*$. Indeed, in expectation ISS results. However, the wages proposed by both players in the SPE of this limiting game are extreme in the sense that they direct the entire surplus to the proposing player, and this is true for each $A > 0$ and does not disappear for $A \downarrow 0$.

This is a highly unattractive feature of the ultimatum game, as it predicts the existence of two extreme wages in the economy, only corresponding to ISS in expectation. This may not seem very relevant given risk neutrality of the agents, but it, for instance, creates strong incentives for search on the job, which is assumed away in most matching models. An obvious alternative arises if we have τ increase without bounds in the limit game of Proposition 1 ($A \downarrow 0$). This is not an ultimatum game, as there will almost surely be another proposal after each rejection for all $\tau > 0$, and it generates ISS in the limiting SPE. Furthermore, in this limiting SPE, ISS does not only hold in expectation, but also independently of the identity of the first mover.

However, the critique that $\tau \rightarrow \infty$ drives a large wedge between the bargaining and production environments remains. It is not clear how we can defend that matches break down infinitely fast during disagreement, given that shocks destroying the productive capacity of the match arrive at a **finite** rate ($a < \infty$) in any nontrivial version of the model. The purpose of our analysis is to show that ISS can be supported without relying on extreme differences between bargaining and producing matches, i.e. for arbitrary breakdown rates τ .

A first step in this direction is made by Rubinstein and Wolinsky (1985), who show that, in a setting similar to ours, ISS can also be supported if agents exert the appropriate search efforts in the disagreement state. In our terminology, if $s_i = s_i^*$, the expected capital gains of encountering market opportunities are the same irrespective whether the unemployed are unmatched or bargaining. Consequently, these gains are internalized in the disagreement payoffs, and ISS arises.

However, as first shown by Wolinsky (1987), if agents at some point choose the search efforts s_i during disagreement, they will in general not pick s_i^* . In Abbring (1997), I discuss an extension of Procedure 1 in which, in each bargaining round n , there is an additional stage between $n.2$ and **$n.3$** in which agents simultaneously choose search efforts for stage $n.3$. Under the assumption that these search efforts are unobserved, i.e. that strategies are not allowed to condition on these search efforts, we end up in a model similar to that of Wolinsky (1987).¹¹

The incentives for search during disagreement are limited, compared to the incentives for unmatched search. Roughly, unemployed of type i gain the difference between $W_i(\bar{w})$ and V_i^* and bargaining agents of type i the difference between $W_i(\bar{w})$ and a slightly discounted $W_i(Ew)$. So, in general agents will exert less search effort while bargaining, and ISS will not arise. Furthermore, in Section 6 we will argue that in market equilibrium

¹¹ Wolinsky (1987) considers a model with stochastic matches, i.e. with ex match heterogeneity. Costain (1996) discusses endogenizing search intensities in a model similar to that of Rubinstein and Wolinsky (1985), and therefore ours.

$\bar{w} = Ew$, so that the only gain of search during disagreement is the avoidance of some discounting by shifting the timing of the payoff $W_i(Ew) = W_i(\bar{w})$ slightly closer. Obviously, this gain disappears as $A \downarrow 0$, in which case agents will not search during disagreement. Then, $s_i = 0$, and wages are given by instantaneous surplus sharing.

The reason for this result is that search for market opportunities has no strategic value, as agents cannot commit to search before disagreement occurs, and search is unobserved. Costain (1996) mentions a variant in which search is observed, but agents cannot commit either. As is known from the literature on bargaining with endogenous disagreement payoffs, such a model typically has a multiplicity of SPE, and is therefore of only limited value to our purposes (Busch and Wen, 1995). In the next subsection we show that ISS can be supported as the unique SPE of a bargaining game in which agents can commit to (observable) search.

5 Bargaining with precommitment to search

Suppose that, before entering the actual bargaining process, agents can precommit to exerting a certain level of search effort in case of disagreement. This can be formalized as

Procedure 2. When just matched, the worker and firm choose (constant) search intensities s_w and s_f respectively. Then, they enter node 0.0 of Procedure 1 and are committed to the selected search intensities.

Commitment of this type, allowing agents to precommit to disagreement actions before bargaining starts, has a long history in bargaining theory, going back to the seminal work by Nash (1953) and Schelling (1960). Note that the agents can precommit to disagreement actions, but only actually incur the corresponding costs in case of disagreement. We return to this issue later.

In the precommitment game, each pair of search intensities (s_w, s_f) selected in the first stage is followed by a bargaining **subgame** similar to the game with exogenous search. Thus, in any SPE of the precommitment game, the SPE of each bargaining **subgame** is the unique SPE of Proposition 1, for the appropriate values of s_w and s_f . This implies that there is again no delay in SPE, and in any SPE of the precommitment game, s_w and s_f should maximize $W_w(Ew)$ and $W_f(Ew)$, respectively, with Ew given by (2). We have the following result.

Proposition 2. *The search and bargaining model following Procedure 2 has a unique SPE in which a bargain is struck without delay. This SPE is characterized by ISS and search efforts that maximize the value of unemployment, or $s_i = s_i^*$, $i = w, f$.*

Proof. Let $\beta_w = \beta$ and $\beta_f = 1 - \beta$. Then, the first order conditions for optimal search s_i are

$$\beta_{j(i)} (c'_i(s_i) - W_i(\bar{w}) + D_i(s_i, s_{j(i)})) = -\beta_i (V_{j(i)}^* - D_{j(i)}(s_{j(i)}, s_i)), \quad i = w, f. \quad (3)$$

First note that (3) is satisfied if $s_w = s_w^*$ and $s_f = s_f^*$.

Next, the l.h.s. of (3) has the sign of $s_i - s_i^*$ and the r.h.s. of (3) is nonpositive, implying that

$$s_w \leq s_w^* \quad \text{and} \quad s_f \leq s_f^*. \quad (4)$$

Finally, suppose that $s_i < s_i^*$. The two conditions (3) can be combined into

$$\beta_f (c'_w(s_w) - W_w(\bar{w}) + V_w^*) = -\beta_w (c'_f(s_f) - W_f(\bar{w}) + V_f^*),$$

so that $s_i < s_i^*$ implies that $s_{j(i)} > s_{j(i)}^*$, which is a contradiction of (4). So, $s_i = s_i^*$ and $D_i = V_i^*$, $i = w, f$. $\square$

The intuition for this result is clear. Recall that agent i maximizes D_i to equal V_i^* by searching with effort $s_i = s_i^*$. So, if agent i precommits to search with effort $s_i = s_i^*$, then $D_i = V_i^*$, independently of $s_{j(i)}$. Then, as agent i is indifferent between disagreement and being unmatched, agent $j(i)$ cannot affect the bargaining outcome via D_i , and optimizes the outcome by simply maximizing $D_{j(i)}$, which yields $D_{j(i)} = V_{j(i)}^*$ and $s_{j(i)} = s_{j(i)}^*$. Obviously this holds for $i = w$ and f alike, so that ISS with $s_i = s_i^*$ can be supported as a SPE of the strategic bargaining game with precommitment.

Also, there are no SPE such that $s_i > s_i^*$ for either $i = w$ or f . To see that this is true, note that $D_{j(i)}(s_{j(i)}, s_i) < V_{j(i)}^*$ for all $s_{j(i)} \neq s_{j(i)}^*$, so that $D_{j(i)}(s_{j(i)}, s_i)$ increases with s_i for all $s_{j(i)} \neq s_{j(i)}^*$. Therefore, given any search effort $s_{j(i)} \neq s_{j(i)}^*$ of agent $j(i)$, agent i can increase $D_i(s_i, s_{j(i)})$ and decrease $D_{j(i)}(s_{j(i)}, s_i)$ by reducing s_i from a level above s_i^* to s_i^* .

Finally, note that a search effort $s_i < s_i^*$ is suboptimal from the perspective of the maximization of D_i , and corresponds to marginal benefits from search (in terms of D_i) proportional to $W(\bar{w}) - D_i = (W(s) - V_i^*) + (V_i^* - D_i) > W(\bar{w}) - V_i^*$. Thus, for s_i to be optimal in the precommitment game, this should be compensated with a sufficiently large marginal effect on $D_{j(i)}$: $V_{j(i)}^* - D_{j(i)}$ should be sufficiently large relative to $V_i^* - D_i$. As both $s_w < s_w^*$ and $s_f < s_f^*$ are required, this should hold for $i = w$ and f simultaneously. The proof shows that this is not possible, so that no SPE exist such that $s_w < s_w^*$ and $s_f < s_f^*$.

This leads to our main conclusion that we can support ISS as the unique SPE of our bargaining game with precommitment to search. This result is derived independently of the value of τ , which is not required to equal σ or to grow without bounds. As argued in Section 4, this is an advantage over the alternative line of defense, which draws on having $\tau \rightarrow \infty$. We conclude by reviewing some issues that deserve additional discussion.

6 Discussion

So far, we have restricted attention to a partial analysis of the bargaining and search game played by a single worker and firm, taking market wages as given. The extension to market equilibrium, in which consistency between market wages and decentralized bargaining outcomes is required, is standard and does not affect the results on ISS found so far (see Wolinsky, 1987). Given that agents are infinitely small, market equilibrium is symmetric, and $\bar{w} = Ew$. This fact can be exploited to derive some additional results. One implication of $\bar{w} = Ew$ is that the outside option of leaving the bargaining game for the unmatched state is never binding in market equilibrium. Thus, we can grant the responding player the additional choice of taking up this outside option without changing the results (Binmore, Shaked and Sutton, 1989). Also, because of discounting, agents will always prefer to take up an encountered market opportunity instead of waiting for the next bargaining round with the current partner, as both yield the same payoff Ew . Thus, the restriction that agents are forced to move into an alternative match when it is contacted has no bite in market equilibrium.

A second issue is the assumption that contracts are not renegotiated. Unlike the Nash solution, which is static and can be imposed at each point time, the strategic approach requires a full specification of the timing of the bargaining game and the type of contracts that are written. In this paper, agents bargain over a contract right after a match has been formed. As soon as agents have agreed upon a contract, they are committed to trade against the wage specified in the contract as long as the match continues. As stated before, this does not lead to inefficiencies, as producing matches are only hit by shocks destroying all production possibilities. If we maintain the assumption that agents are committed to trade after the initial contract has been written, we can easily extend our bargaining game into a renegotiation game as in Macleod and Malcomson (1993, 1995). As commitment to trade excludes strikes and lockouts, and thus ensures exogenous disagreement payoffs, the renegotiation game has a unique SPE in which the same initial contract is written and never renegotiated. In a model in which matches are hit by more types of shocks than just fatal productivity shocks, like non-fatal productivity shocks or shocks affecting the outside options of agents, we would still have a unique SPE, in which the contracts are renegotiated whenever the outside option of either agent is binding. If we could make the more realistic assumption that contracts only specify wages, and cannot force agents to trade, our renegotiation game would be a bargaining game with endogenous disagreement payoffs. Such a model is known to have multiple SPE, which makes this approach less suitable for the purpose of this paper (Macleod and Malcomson, 1995).¹²

¹²More in general, the multiplicity of SPE in sequential bargaining games with endogenous disagreement payoffs is well known. See Haller and Holden (1990), Fernandez and Glazer (1991) and Busch and Wen

We conclude by shortly discussing the commitment assumption. In our model, agents can **precommit** to search without incurring any of the corresponding costs. Although this is fairly standard, one may prefer a slightly subtler version of the model, in which agents can only commit to search one bargaining period ahead, and have to incur the corresponding expected search cost even if agreement is reached before search occurs. More precisely, instead of allowing agents to choose search intensities for the entire bargaining game before node 0.0, as in Procedure 2, we could allow agents to choose search intensities for state $n.3$ between $(n-1).3$ and $n.0$ in each period n . We could then require agents to pay for the expected search costs even if stage $n.3$ is never reached, but in any equilibrium without nontrivial delay this would be irrelevant to the analysis in the limit $A \downarrow 0$. The model set up in this manner is closer to a version of the model by Wolinsky (1987) with commitment to observable search than the model in this paper. Thus, it would be interesting to characterize the set of SPE in such a model, and compare these to the set of SPE in a model with observable search efforts without **commitment**.¹³ However, as this is a nontrivial extension of the model that would draw attention away from the simple point made in Proposition 2, we leave this for future research.

Appendix: Proof of Proposition 1 (sketch)

Let $\alpha := \tau + s_w + s_f + p$. In any SPE, the worker and firm proposals w^w and w^f should satisfy

$$W_i(w^{j(i)}) = \frac{1 - e^{-\alpha\Delta}}{\alpha} \{ [\tau + s_{j(i)}]V_i^* + s_i W_i(\bar{w}) + \zeta_i - c_i(s_i) \} + e^{-\alpha\Delta} W_i(\mathbb{E}w), \quad (5)$$

for $i = w, f$. The arguments provided by Shaked and Sutton (1984) can be used to prove uniqueness of the SPE. Next, note that the two SPE conditions in (5) imply that

$$W_w(w^f) - W_w(w^w) = \frac{1 - e^{-\alpha\Delta}}{\alpha} \left\{ -\alpha \bar{W} + \sum_{i=w,f} [\tau + s_{j(i)}]V_i^* + s_i W_i(\bar{w}) + \zeta_i - c_i(s_i) \right\},$$

and therefore $\lim_{\Delta \downarrow 0} w^w - w^f = 0$. Finally, the SPE conditions in (5) can be combined into (2).

(1995). A more extensive discussion on assumptions on contracting and bargaining issues in a matching model can be found in Abbring (1997).

¹³Note that Wolinsky (1987) avoids all problems by assuming that search is unobservable.

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