

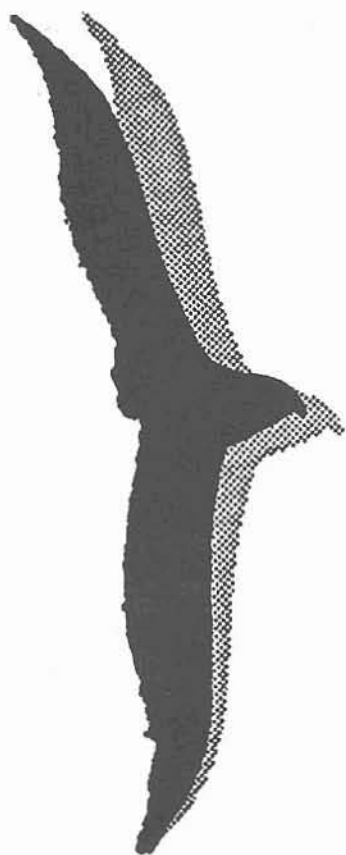
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Job Reallocation, Idiosyncratic Shocks and Aggregate Fluctuations

Pieter Gautier

Research Memorandum 1996-32

July 1996



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Abstract:

This paper shows how idiosyncratic shocks, macro-economic complementarities and a reallocation timing effect, can lead to multiple cyclical output equilibria. When reallocating labor from low productivity plants to high productivity plants takes time and effort which cannot be used for "normal production", the best time for an individual firm to reallocate labor is in a recession when demand is low anyway. In the presence of complementarities and positive spillovers, a negative demand shock for one firm will give this firm an incentive to reallocate, but because reallocation itself also leads to lower output this will decrease the demand for the goods of other firms. Therefore those other firms will have an incentive to reallocate as well. After the reallocation process is finished, output will increase again. (JEL classification: D70, E32, J20, J60, Keywords: business cycle, coordination failures, reallocation)

Correspondence to: Tinbergen Institute, Keizersgracht 482, 1017 EG Amsterdam, The Netherlands, fax: 3120-551-3555, email: gautier@tinbinst.nl.

1 Introduction

In the last decade economists have become more and more convinced that coordination failures are an important factor to take into account when it comes to explain the fluctuations in output and unemployment. A particularly interesting model with coordination failures is Shleifer's (1986) implementation cycle model. Shleifer argues that given the fact that an innovation is profitable for a limited period of time (until it is copied by competitors), firms may store inventions and implement them at times when aggregate demand is high. If firms hold commonly shared expectations about the future path of the economy, they will choose an independent pattern of investment that will make those expectations come true. Moreover, Shleifer shows that under certain conditions, any expectation about the date of a boom will lead to that particular boom.

A problem with Shleifer's model is that its predictions are not in line with the data. Recently a number of authors e.g. Davis and Haltiwanger (1990, 1992) Blanchard and Diamond (1990) for America, Broersma and Gautier (1997) for the Netherlands, and Burda and Wyplosz (1994) for a number of other European countries, have studied the cyclical behavior of job and worker flows. They all found that booms are not times with high rates of job creation, as Shleifer's model would predict, but rather times when job destruction is low. Moreover, Bean (1990) and Saint-Paul (1993) showed that a positive demand innovation has a negative effect on future productivity growth.

But the idea that if firms have commonly shared expectations of the business cycle, those expectations may well come true, remains interesting. We will therefore modify Shleifer's story in such a way that not the (common) expectations about booms but the (common) expectations about recessions cause the cyclical behavior of output and employment.

Endogenous timing of reallocation activities is a crucial factor in our model. The idea is simple. If we view reallocation as a process in which unprofitable jobs are destroyed and profitable jobs are created, then given the fact that reallocation takes time and effort which cannot be used for normal production, the best time to reallocate is when the opportunity costs (in terms of foregone production) are lowest. This is the case in recessions when aggregate demand is below its normal level.

The paper is organized as follows. First in section 2, we will work out a simple model of job reallocation and we will look what will happen in a perfect foresight world, when firms cannot coordinate their actions. If reallocation is cheapest in recessions and if all firms have the same expectations about the date and the size of a recession, they might time their reallocation policies simultaneously in a recession, thereby making this recession a reality. Then in section 3, we will give

a numerical example. We will show that cyclical fluctuations of output are not necessary bad cause it gives firms the opportunity to "seed" in good times and to "harvest" in bad times. Finally, section 4 concludes.

2. A model of Labor Reallocation

In this section we will work out a model of labour reallocation. We are particularly interested in the question: "What is the best time to reallocate labor from low to high productivity plants, given the fact that this process takes time.?" It will become clear that this model exhibits both complementarities and positive spillovers. A nice feature of the model is that it describes a mechanism how allocative shocks interact with aggregate shocks.

2.1 The assumptions

The model is an extension of Davis and Haltiwanger's (1990) prototype model of job reallocation. In their model, aggregate shocks drive the business cycle but because of allocative frictions, especially the negative shocks can get amplified. We will show that in the presence of complementarities, idiosyncratic shocks which hit only one or a few firms can lead to large aggregate fluctuations as well.¹ The model describes an economy with two types of production sites, high productivity sites (Highs) and low productivity sites (Lows). At the beginning of period t , a fraction H_t of the workers is matched with a high productivity site and the rest of the workers, a fraction $(1-H_t)$ is matched with a low productivity site. High productivity sites do not remain high productivity sites forever. Each period a fraction σ of the Highs becomes a Low, for example because of depreciation. It is however possible to transform a low productivity site to a high productivity site at the costs of one period of foregone production. The fraction of lows that become high will be denoted with Θ . There are a number of interpretations we can give to those costs: The first is that the firm has to fire low productivity workers and hire high productivity workers and that this process takes time. Or from the workers point of view, that workers have to quit their job at a Low and search a period to find a job at a High. A second interpretation is related to the Kydland and Prescott's (1982) concept of "Time to build". It takes one period of low productivity worker input to open a High, during that period this worker can not be used for "normal" production activities. Finally we can interpret those costs as an investment in human capital by the firm and the worker. All of this is captured in the following law

¹ This in line with the empirical observations of Davis and Haltiwanger (1990) and Abbring and Gautier (1996) that most of the job reallocation is driven by idiosyncratic shocks and not by aggregate shocks.

of motion, which is assumed to hold for all firms:

$$H_{t+1} = (1 - \sigma_t)H_t + \Theta_t[1 - H_t + \sigma_t H_t] \quad (1)$$

Accordingly we will define the production functions $f(a_H)$ and $f(a_L)$ to be equal to the output of working a hours in respectively high productivity and low productivity plants. The value of output in some firm i , varies with demand from other firms $j \neq i$. This is where the complementarities come in. When all firms $j \neq i$ increase their output, the demand for the product of firm i will also increase. This will increase the price for good i (which we did not explicitly model²) and will finally increase firm i 's output. We will denote the aggregate demand of the other firms by A . An other interpretation is that A is a "value of production" shifter. Thus the output of a high productivity worker who works a hours is $Af(a_H)$.³ To sum up: each firm has heterogeneous jobs, and all firms are heterogeneous in their output as well, but they are homogeneous in the way they reallocate labor over the different jobs. This is captured in the following dynamic programming problem where each firm maximizes total output from both Highs and Lows:

$$V(H, \sigma, A) = \underset{a_H, a_L, \Theta}{\text{Max}} A[(H - \sigma H)f(a_H) + (1 - H + \sigma H)[(1 - \Theta)f(a_L) + \beta EV(\bar{H}, \bar{\sigma}, \bar{A})] \quad (2)$$

and the law of motion (1).⁴

The weights are given by: $(1 - \sigma)H$ (the number of workers in the high productivity sector), $(1 - H + \sigma H)(1 - \Theta)$ (the number of workers in the low productivity sector) and $(1 - H + \sigma H)\Theta$ workers in the search state mode. The first order condition with respect to Θ implies that:

$$A(1 - \sigma)f(a_L) = \beta E \left[\frac{\partial V(\bar{H}, \bar{A}, \bar{\sigma})}{\partial \bar{H} | A, \sigma} \right] \quad (3)$$

The first term of the l.h.s. of equation (3) represents the foregone output that results from reallocating labor. The r.h.s. of (3) gives the expected utility gains resulting from an improved future allocation of labor. Thus at an interior solution for Θ , the total utility of improved future allocation is equal to the current output loss due to reallocation.

² This could be done in a monopolistic competition setting as in: Blanchard and Kiyotaki (1985) and Startz (1989).

³ One could think of a situation in which the output of one firm is used as input for the other firms, see e.g. Cooper and John (1993a).

⁴ We will not proof the existence of a unique optimal policy function $\Theta(H, A, \sigma)$ here. Instead we refer to Lucas et al. chapter 9.

What will happen with employment outflow and job reallocation when demand (A) falls? First note that employment outflow is given by:

$$\begin{aligned} E_{out} &= \Theta(1-H+\sigma H) \quad \text{and} \\ \frac{\partial E_{out}}{\partial H} &= -(1-\sigma)\Theta = \frac{\partial \Theta}{\partial H} - (1-\sigma) \end{aligned} \tag{4}$$

We can see directly from equation (4) that in recessions (A falls), there will be more reallocation activity, since when A is low the costs of reallocation in terms of foregone production are lower. The increase of Θ will first increase job destruction and then in the next period job creation will rise. This is consistent with the evidence from Davis and Haltiwanger (1990) and Gautier and Broersma (1994).

Sofar the model is still very similar to the one in Davis and Haltiwanger (1990). We will now follow Shleifer (1987) who considered a perfect foresight world and focused on constant period symmetrical cycles of period T . In our model, cycles are caused by fluctuations in A , T is then the time between two maxima or minima of A . There are two conditions that have to be met for T to be a perfect foresight cyclical equilibrium. First recall that firms can reallocate labor any time they like. One condition would be then that firms do not reallocate before T . The second condition is that if firms expect a recession in period T , they will not wait with reallocating, till after T ⁵.

It is easy to see that if firms do not reallocate before or during a recession, they will not reallocate at all. This situation can be viewed as a "stone age equilibrium"⁶. The reason for this is that if reallocating after the first expected recession would have been profitable, reallocating in the recession would have been more profitable (since we assumed constant period cycles). Even reallocating before the first expected recession would have been more profitable, since we discount future profits. So either firms will reallocate in the first expected recession or before that recession.

A firm would reallocate before T , if the cost of waiting till the first recession arrives exceeds the

⁵ An important question which is often neglected is why firms would have common expectations? It is always easy to explain an economic event by saying: "It happened, cause people expected it to happen". Keynes himself emphasized with his famous "beauty contest" metaphor that firms spend more resources predicting: "what average opinion expects the average opinion of the economy to be" than trying to figure out what the true state of the economy is. Woodford (1991) points out that small random events that change people's expectations and which do not affect the fundamentals make it rational for all people to change their expectations. The resulting equilibria are sometimes called sunspot equilibria.

⁶ This can happen when the difference between f_{aH} and f_{aL} is very small.

benefits of reallocating in a recession. In the appendix, it will be shown that when a firm waits with reallocating till the first recession at period T , its output will be equal to:

$$\begin{aligned} \sum_{i=1}^{T-t} Q_{t,i} = & \left[Af(a_H)H_t \frac{1-\beta^{T-t+1}(1-\sigma)^{T-t+1}}{(1-\sigma)^2} \right] + (1-H_t)Af(a_L) + \\ & \frac{\sigma H_t Af(a_L)}{\beta(1-\sigma)} \left[\frac{1-\beta^{T-t+1}(1-\sigma)^{T-t+1}}{1-\beta(1-\sigma)} - (T-t+1)\beta^{T-t+1}(1-\sigma)^{T-t+1} \right] - \\ & \theta\beta^{T-t} \left[1 - H_t + \sigma H_t Af(a_L) \frac{1-(1-\sigma)^{T-t+1}}{1-\sigma} \right] \end{aligned} \quad (5)$$

A firm will innovate at T , when ΣQ exceeds ΣQ^* , the output that is obtained when the firm would not have postponed its reallocation activities to a recession. ΣQ^* is given by:

$$\begin{aligned} \sum_{i=1}^{T-t} Q^*_{t,i} = & A \max_{\Theta} \left[\sum_{i=0}^{T-t} \beta^i [(1-\sigma)H_{t+i}f(a_H) + (1-\Theta)(1-H_{t+i} + \sigma H_{t+i})f(a_L)] \right] \\ \text{s.t. A law of motion: } & H_{t+1} = (1-\sigma)H_t + \Theta_t[1 - H_t + \sigma H_t] \end{aligned} \quad (6)$$

The solution of this dynamic programming problem gives the optimal reallocation policy of a firm that ignores the behaviour of other firms and hence aggregate demand (A). If workers who are in the search (or training) state have less income and will consume less goods, the recession will become more severe and the incentives to concentrate the reallocation activities in a recession will be even larger. We will abstract here from labor supply decisions.

If the recession is expected the next period, firms will obviously reallocate in that period. If the recession is expected to arrive in period $t+2$, firms will wait with reallocating till period $T=t+2$ if the advantages of waiting (lower opportunity costs of foregone output), exceed the losses (less high productivity sites in period $t+1$). The same reasoning holds for $T=t+3 \dots t+n$. Thus any time interval $(T-t)$ that satisfies:

$$\sum_{j=1}^{T-t} Q_{t,j} \geq \sum_{i=1}^{T-t} Q^*_{t,i} \quad (7)$$

can lead to a recession in period T , depending on the expectations when this recession will take place. If (7) would hold for any time interval $(T-t)$ (which is not very plausible), the number of possible equilibria would be infinite. Any possible equilibrium would then only depend on the (common) expectations of the date the recession would take place.

Besides complementarities, this model will also give rise to positive spillovers between firms if for example an increase in firm j 's output benefits consumers of that product. Cooper and John (1988) show that when there are strategic complementarities, positive spillovers globally and the slope of the reaction curve is greater than one, that there are multiple Nash equilibria that can be Pareto ranked, by equilibrium action (where higher action equilibria are preferred).

In the next section, we will work out a small numerical example to understand the working of the model better.

3.1 An example

We will now illustrate the model of the previous section with a simple numerical example. First we will consider five possible cycles, then we will ask which of those cycles fulfils the property of a perfect foresight cyclical equilibrium for certain parameter values of H_0 , σ , A , $f(a_H)$ and $f(a_L)$. Furthermore we will distinguish two reallocation strategies for an individual firm. The first strategy is to wait with reallocation till the first recession takes place and then choose that value for Θ which maximizes the discounted value of output. The second strategy is to choose an optimal value for Θ without taking the behaviour of other firms into account. We will represent a cycle as follows: In normal times, the value of the aggregate demand parameter, A is equal to 1 while in recessions A is 0.5.

What will be the best strategy for a firm when all other firms expect a recession at $T=2$? Let $T2W$ be the strategy: *Wait with reallocation till the first recession (every second period) and then choose the optimal value for the reallocation parameter Θ* and let $T2R$ be the strategy: *Choose an optimal value for Θ , independent of the reallocation policies of other firms, which are reflected in A (or: do not concentrate reallocation in recessions)*. In table 1 a number of different reallocation strategies are presented.

table 1 Output under different reallocation strategies

Period	T2W	T2R	T3W	T3R	T4W	T4R	T5W	T5R
1	<u>0.94</u>	<u>1.14</u>	2.50	2.27	<u>0.94</u>	<u>1.14</u>	1.80	2.27
2	4.75	4.12	<u>0.7</u>	<u>2.06</u>	4.76	4.12	1.58	4.12
3	<u>1.70</u>	<u>1.98</u>	4.76	3.96	3.80	3.96	1.37	3.96
4	4.76	3.8	3.79	3.80	3.07	3.85	1.47	3.86
5	<u>1.72</u>	<u>1.82</u>	<u>1.28</u>	<u>1.82</u>	<u>0.955</u>	<u>1.82</u>	<u>0.295</u>	<u>1.82</u>
6	4.76	3.47	4.76	3.47	4.76	3.47	4.75	3.47
$\Sigma B^i Q$	16.14	14.26	15.43	15.23	15.98	16.05	11.4	17.18

$H_0=0.5$, $\sigma=0.25$, $f(a_h)=1.5$, $f(a_l)=0.5$, $\beta=.95$, $A=0.5$ in period T_i (recessions) and 1 in other periods. Underlined numbers represents output in recessions. For each strategy, Θ is chosen to optimize output over a six period time span.

In this specific example it pays to wait for $T = 1, 2$, and 3 cycles but when a recession is expected every fourth or fifth period, postponing the reallocation activities does not pay any more, the discounted value of output is lower when the firm concentrates its reallocation activities in a recession.

Figure 1 shows that if the firm decides to concentrate reallocation in recessions (strategy T2W), output is slightly lower in recessions than under strategy (T2R), but it is much higher in booms.

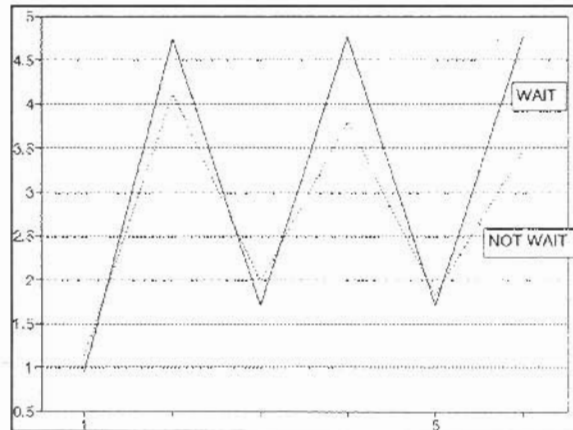


Figure 1 T=2 Cycle

So if all other firms decide to postpone reallocation till a recession this is also the optimal strategy for the last firm, hence $T=2$ is a perfect foresight cyclical equilibrium.

Figure 2 shows the case for a $T=3$ cycle. Waiting with reallocation till a recession results in a higher discounted value of output than reallocating every period. We also see that the $T3W$ cycle has a larger amplitude than the $T3R$ cycle because reallocation (which in the $T3W$ case takes place in a recession) itself leads to lower output in the first period and higher output in next periods.

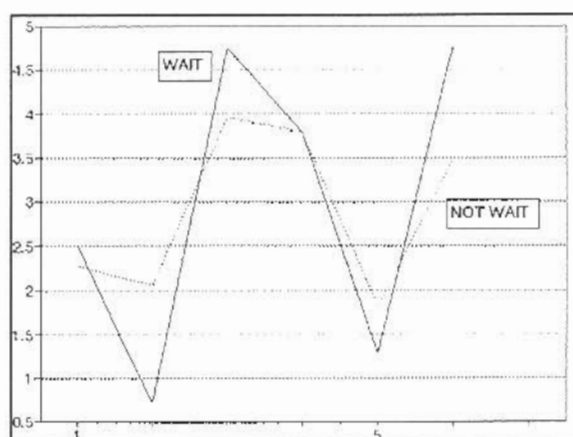


Figure 2 $T=3$ cycle

For $T>4$ -cycles "waiting" no longer pays Figure 3. The costs of waiting (a sub optimal allocation during more then 4 years) exceeds the benefits of waiting (cheaper reallocation). So the $T=4$ -cycle cannot be a perfect foresight cyclical equilibrium in our example.

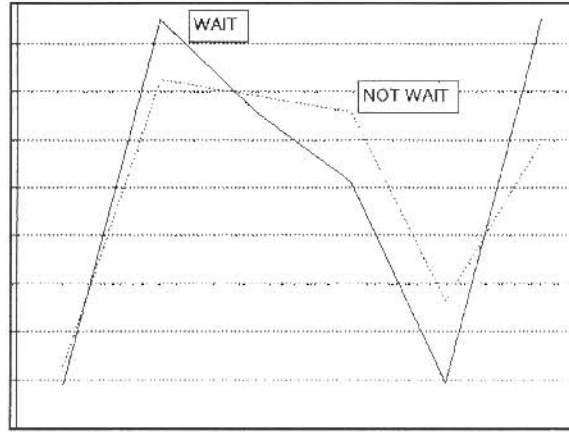


Figure 3 T=4 Cycle

It is also interesting to see how parameter changes influence the allocation decisions. For example, an increase in the spread between $f(a_H)$ and $f(a_L)$ does not change any of the results qualitatively. This may at first be somewhat surprising since one might expect that an increase in the spread will make "waiting" more costly, but what happens is that the chosen value for Θ just increases and the costs for this increase just happen to be lower in a recession. In an environment in which allocating is relatively unfavorable because high productivity sites revert quickly to low productivity sites ($\sigma = .75$), it still pays to postpone reallocation but the optimal value for Θ falls from 0.95 to 0.3.

3.2 The "seed" role of recessions

In the examples described above, we made the simplifying assumption that recessions are times when the value of output is below its "normal" value. If we would also allow for times when the value of output is above its "normal" level, we could check if there are cases when recessions play a useful "seed" role. Thus in the context of our model: If we would consider two series for $Af(a_H)$ and $Af(a_L)$, of which the sum is equal but one is cyclical and the other one is constant over time and it would be the case that the cyclical series generate a higher production than the constant series because more workers will be allocated to Highs, we can conclude that a zero variance of output is not optimal. Figure 4 shows that this situation is possible in our model⁷. Output under the "wait" strategy is strongly procyclical. Total output exceeds output in the case of constant $Af(a_H)$ and $Af(a_L)$.

⁷ Parameters are set at the following values: $Af(a_H) = 20$, $Af(a_L) = 10$, $\sigma = 0.75$, $H_0 = 0.5$, $\beta = 0.95$. For the cyclical series, the following holds: In booms $Af(a_L) = 10.5$ (they last two periods) and in recessions $Af(a_L) = 9$, they last one period.

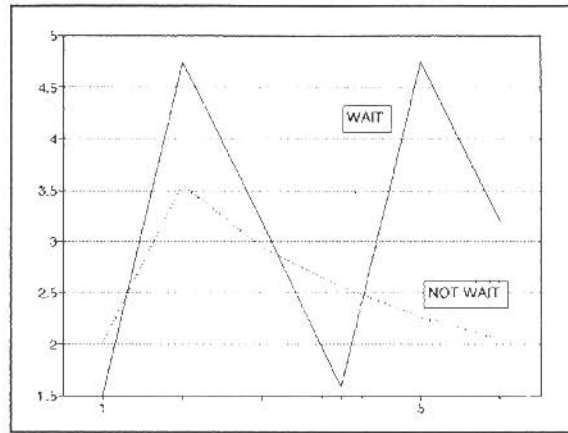


Figure 4 Constant and cyclical A

We see thus that in this model, that some variance in the output per worker (while the mean output value remains the same) for both job types can lead to higher total output because more high productivity jobs can be created. At the same time, recessions will be more severe and in booms output will even be higher.⁸ We also see that at the constant value of output series, there will only be reallocation from Highs to Lows in the first period, after that it will not be optimal to open up more Highs at the expense of current production.

4 Final Remarks

Traditionally, both supply and demand shocks and cyclical and structural shocks have often been considered to be different things and were believed to have different effects.

In this paper we showed that demand shocks, which are often assumed to have only cyclical effects, can also have structural effects and affect the supply side of the economy. The mechanism goes through a reallocation timing effect. Firms have an incentive to concentrate the time consuming process of reallocation in times when the demand for their product is low because the opportunity costs of doing so are lower then.

On the other hand, we have showed that "structural shocks" can influence the demand side of the economy when the reallocation process itself leads to a temporary fall in output. This interaction between demand and supply shocks can generate a cyclical movement of output. We have also shown that this cyclical movement is not necessary bad. Cyclical demand gives firms the opportunity to

⁸ We abstracted here from possible imperfections in the demand for credit, which could lead to a stream of bankruptcies and this could change the conclusions of the model.

"seed" in bad times and "harvest" in good times. Stabilizing fiscal and monetary policies can therefore not only dampen the business cycle but also economic growth, even if it is timed correctly.

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Appendix:

Derivation of equation (5)

Under the assumption that σ_t is the same and equal to σ , for all t in the interval $[t, T]$, and when there is no reallocation, from t till T ($\Theta_1 \dots \Theta_{T-1} = 0$, $\Theta_T = \Theta$). Output in high productivity sites and low productivity sites, is given by:

period	High Productivity sites	Low Productivity Sites
t	$(1-\sigma)H_t Af(a_H)$	$\{1-H_t + \sigma H_t\} Af(a_L)$
t+1	$\{(1-\sigma)(1-\sigma)\}H_t Af(a_H)$	$\{1-H_t + \sigma(1+(1-\sigma))H_t\} Af(a_L)$
t+2	$(1-\sigma)(1-\sigma)(1-\sigma)H_t Af(a_H)$	$1-H_t + \{\sigma(1+(1-\sigma)+(1-\sigma)(1-\sigma))\}H_t Af(a_L)$
....		
....		
....		
T	$(1-\sigma)^{T-t} H_t Af(a_H)$	$(1-\Theta)(1-H_t + [\sigma \sum_{i=0}^{T-t} (1-\sigma)^i H_t] Af(a_L))$
total	$\sum_{j=1}^{T-t} (1-\sigma)^j H_t Af(a_H)$	$(T-t-\Theta)H_t + \sigma \sum_{i=0}^{T-t} (T-t-i-\Theta)(1-\sigma)^i H_t Af(a_L)$

Thus total output in period T, equals:

$$\sum_{j=1}^{T-t} (1-\sigma)^j Af(a_H) H_t + \left[(T-t)(1-H_t) + \left[\sigma \sum_{i=0}^{T-t} (T-t-i)(1-\sigma)^i \right] H_t - \Theta \left[1-H_t + \sigma \left[\sum_{i=0}^{T-t} (1-\sigma)^i \right] H_t \right] \right] Af(a_L)$$

This equals: $\sum_{j=1}^{T-t} (1-\sigma)^j H_t f(a_H) + [(T-t-\sigma)(1-H_t) + \sigma \sum_{i=0}^{T-t} (T-t-i-\Theta)(1-\sigma)^i] H_t Af(a_L)$

The value of output between period t and T is equal to the value of the sum of all high and low productivity output in each period t, ..., T. This results in:

$$\begin{aligned} & \sum_{j=1}^{T-t} \beta^j (1-\sigma)^j Af(a_H) H_t + (1-H_t) Af(a_L) + \left[\sigma \sum_{i=0}^{T-t} \sum_{j=i}^{T-t} \beta^j (1-\sigma)^j \right] H_t Af(a_L) \\ & - \Theta \beta^{T-t} \left[1-H_t + \sigma \left[\sum_{i=0}^{T-t} (1-\sigma)^i \right] H_t \right] Af(a_L) \end{aligned} \quad (A1)$$

Since a geometric series given by:

$$\sum_{j=0}^{n-1} ar^j = a \frac{1-r^n}{1-r} \quad (A2)$$

and

$$\begin{aligned} & \sum_{j=0}^n \sum_{i=j}^n r^i = \sum_{j=0}^n \left[\sum_{i=0}^n r^i - \sum_{i=0}^{j-1} r^i \right] \\ & = (n+1) \frac{1-r^{n+1}}{1-r} - \frac{1}{1-r} \sum_{j=0}^n 1-r^{n+1} = (n+1) \frac{1-r^{n+1}}{1-r} - \left[\frac{1}{1-r} (n+1) - \frac{1-r^{n+1}}{(1-r)^2} \right] \\ & = \frac{1}{1-r} \left[\frac{1-r^{n+1}}{1-r} - (n+1)r^{n+1} \right] \end{aligned} \quad (A3)$$

We can write:

$$\begin{aligned} & \sum_{j=1}^{T-t} \beta^j (1-\sigma)^j Af(a_H) H_t = \left[Af(a_H) H_t \frac{1-\beta^{T-t+1}(1-\sigma)^{T-t+1}}{(1-\sigma)^2} \right] \\ & \sigma \sum_{i=0}^{T-t} \sum_{j=i}^{T-t} \beta^j (1-\sigma)^j = \frac{\sigma}{\beta(1-\sigma)} \left[\frac{1-\beta^{T-t+1}(1-\sigma)^{T-t+1}}{1-\beta(1-\sigma)} - (T-t+1) \beta^{T-t+1} (1-\sigma)^{T-t+1} \right] \\ & \sigma H Af(a_L) \sum_{i=0}^{T-t} (1-\sigma)^i = \sigma H Af(a_L) \frac{1-(1-\sigma)^{T-t+1}}{1-\sigma} \end{aligned} \quad (A4)$$

substitution in (A1) yields:

$$\begin{aligned}
& \left[Af(a_{\mu})H_t \frac{1-\beta^{T-t+1}(1-\sigma)^{T-t+1}}{(1-\sigma)^2} \right] + (1-H_t)Af(a_t) + \\
& \frac{\sigma H Af(a_t)}{\beta(1-\sigma)} \left[\frac{1-\beta^{T-t+1}(1-\sigma)^{T-t+1}}{1-\beta(1-\sigma)} - (T-t+1)\beta^{T-t+1}(1-\sigma)^{T-t+1} \right] - \\
& \theta \beta^{T-t} \left[1 - H_t + \sigma H_t Af(a_t) \frac{1-(1-\sigma)^{T-t+1}}{1-\sigma} \right]
\end{aligned} \tag{A5}$$