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Abstract

The alternating offer model is extended to analyse bargaining situations where agents bargain over how to play an infinitely repeated game. Agreements are assumed to be binding. The strategy space is too complex to derive a general characterisation result. However, for the prisoners' dilemma we obtain the classic Folk-Theorem, while for the battle-of-the-sexes game we do not. This leads to the conclusion that the inclusion of bargaining with binding contracts in the repeated game model may reduce the set of SPE payoffs predicted by the Folk-Theorem.

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1 Introduction

Consider an economic situation where two (or more) economic agents interact strategically with each other infinitely often, and, simultaneously, have the opportunity to negotiate over some joint policy which will be implemented if agreed upon (for example, all countries compete with each other on the world market and at the same time negotiate at the GATT). In this context the negotiation process and the strategic interaction simultaneously continue as time proceeds. The consequences for the agents are that each agent not only has to select a strategy in the negotiation process, but at the same time has to decide what strategy to follow in the strategic interaction as long as no agreement is reached. The strategic interaction is modelled as an infinitely repeated static game[1] and an agreement specifies a joint policy of how to play this infinitely repeated game (to refer to the example given earlier, one joint policy could be that all countries stop levying a tax on imports). It is assumed that agreements are *binding* and have an infinite length. Furthermore, attention is restricted to economic situation with only two agents.

In order to model the idea that players have the opportunity to bargaining among each other, not only before the repeated game starts, but especially during the repeated game, the alternating offer model with discounting (Rubinstein [14]) is extended so that the two players have to play the static game in between the bargaining rounds of the alternating offer model. This model, without the assumption of discounting, has been analysed by Okada [12]. Related models are those of Bush and Wen [2], Fernandez and Glazier [8], Haller and Holden [10] and, Houba and de Zeeuw [11]. For a discussion of related work we refer to section 7.

To be more explicit about the bargaining model considered in this paper, each period consists of three stages. In the first stage one of the players proposes a contract, where a contract has to be seen as an infinite sequence of actions in the infinitely repeated (static) game. At the second stage the other player either accepts or rejects this proposed contract. If the proposed contract is rejected, then the static game at the third stage has to be played once by both players before play proceeds to the next period. In this situation each player selects his/her actions at the third stage unilaterally. These actions are called the disagreement actions. As mentioned before, the strategies in this bargaining game not only involve the bargaining behaviour of each agent but also the disagreement actions of each agent. This means that the disagreement actions chosen by a player at a certain period may depend upon the history of bargaining behaviour itself and/or may influence the bargaining behaviour of the other player in the future.

In case the proposed contract is accepted it becomes binding. The bargaining model in this paper is rich enough to allow for renegotiation in the true sense of the word, namely players continue bargaining for some new contract, while the last contract accepted in the past specifies the disagreement actions as long as the players do not agree upon such a new contract. Given that the disagreement actions are fixed in such a subgame the model reduces to the standard alternating offer model [14] with exogenous status quo payoffs, which yields a unique subgame perfect equilibrium (SPE) outcome, namely immediate agreement upon a Pareto efficient contract. Thus, any inefficient contract agreed upon will be replaced by some new and efficient contract. Therefore, the renegotiation process will be disregarded in this paper by assuming that players can only propose efficient contracts.

The bargaining model in this paper always has at least one stationary SPE in which both players use one of the Nash equilibria (NE) of the static game as their disagreement actions and bargain according to the stationary SPE of the alternating offer model [14] as if the disagreement actions were exogenously given. It will be shown that, as the (common) discount factor tends to 1, each player can guarantee himself a minimum payoff that may be higher than this player's minmax value. This implies that the limit set of SPE payoffs need not necessarily be the whole set of individually rational payoffs. For a large class of static games (as in examples 6.4 and 6.5) this convergence result immediately implies that bargaining with binding contracts strictly reduces the set of SPE payoffs predicted by the Folk-Theorem (Fudenberg and Maskin [9]). However, the prisoners' dilemma (example 6.1) shows that a full Folk-Theorem can also be obtained.

The strategy space of the bargaining model in this paper is very complex and it seems not possible to derive a full characterisation result for the set of SPE payoffs. To show this, two types of strategies are defined, namely what we call "immediate agreement" strategies and "one sided offer" strategies. For each type of strategy a "full" characterisation result will be given as well as a procedure to determine the "extreme" SPE payoffs attainable with such a type of strategies. The characterisation result for "immediate agreement" strategies generalises a characterisation result of

Bush and Wen [2] to the bargaining game in this paper. The characterisation result for the "one sided offer" strategies is interesting because there is a relation between these strategies and weakly renegotiation proof equilibria of standard repeated games (Farrell and Maskin [7]). Each of these two characterisation results determines a set of SPE payoffs. Examples 6.2 and 6.3 show that it is not possible to prove that one of these two sets always contains the other set.

The paper is organized as follows. In section 2 the bargaining model will be formally described. In section 3 it is shown that the bargaining model always has at least one stationary SPE. Hence, the set of equilibria will never be empty. In section 4 the characterisation result for the "immediate agreement" strategies is derived, while in section 5 the characterisation result for the "one sided offer" strategies will be given. In section 6 five important examples are given. Section 7 discusses related work, while in section 8 some conclusions are drawn.

2 The Model

The bargaining model in this paper is a dynamic non-cooperative two-player model in discrete time. This model is an extension of the alternating offer model (Rubinstein [14]). Each period t, $t \in \mathbb{N}$, consists of three stages $\theta = 1, 2, 3$. In each third stage the players have to play a static game, denoted by Γ. At the first stage of each period one of the players makes a proposal, which is either accepted or rejected by the other player at the second stage. Without loss of generality it is assumed that player 1 is the player who proposes when t is even, which implies that player 2 proposes when t is odd. See figure 2.1 for an illustration of the game tree.

t even	stage 1	Player 1 proposes a contract.
	stage 2	Player 2 accepts/rejects.
	stage 3	Both players play Γ.
t odd	stage 1	Player 2 proposes a contract.
	stage 2	Player 1 accepts/rejects.
	stage 3	Both players play Γ.

Figure 2.1 The game tree of the bargaining model.

A proposal made at time t, $t \in \mathbf{N}$, specifies a path of actions in the infinitely repeated game, denoted by Γ^∞, from period t onward. Thus, each proposal specifies an infinite stream of payoffs for both players. With respect to a proposal the following assumption is made.

Assumption 2.1 *A proposal is a contract of infinite length that becomes binding once agreed upon.*

To evaluate a stream of payoffs players discount the expected value of this stream. Discounting only takes place after the third stage of each period and before the beginning of the next period. Formally, given player i's stream of (expected) payoffs $\{x_i(\tau)\}_{\tau=0}^{\infty}$ the normalised payoff for this player given the common discount factor δ, $\delta \in (0,1)$, from period t, $t \in \mathbf{N}$, onwards is

$$(1-\delta)\ \Sigma_{\tau=t}^{\infty}\ \delta^\tau x_i(\tau).$$

In what follows below we assume that players can use mixed strategies in the static game Γ. With respect to the observability of mixed actions the following assumption is made.

Assumption 2.2 *Mixed actions are observable.*

The Static Game Γ.

The static game Γ is a two-player normal form game, which is defined formally as

$$\Gamma \ = < \{1,2\},\ \{\ S_i\ \}_{i=1,2},\ \{R_i(s)\}_{i=1,2} >,$$

with $\{1,2\}$ the set of players, S_i the *compact* and *convex* set of (mixed) actions and player i's payoff function $R_i : S \rightarrow \mathbf{R}$, which is *continuous* and *quasi concave* in s_i, given the (mixed) actions $s \in S := S_1 \times S_2$. These assumptions are sufficient for the existence of (possibly mixed) Nash equilibria (NE) in Γ. The set of attainable payoffs in Γ is denoted by $R(S)$. Formally,

$$R(S) = \{x \in \mathbf{R}^2 \mid \exists s \in S \quad \text{s.t.} \quad x = R(s)\},$$

where $R(s) = (R_1(s), R_2(s))$. The convex hull of $R(S)$ is denoted by $R(C)$. Because $R(S)$ is compact, $R(C)$ is also compact.

The minmax value v_i for player i ($i = 1, 2$) and the corresponding pair of (mixed) actions, denoted by $m^i \in S$, are given by

$$v_i = \min_{s_j \in S_j} \max_{s_i \in S_i} R_i(s) \quad \text{and} \quad m^i = \arg\min_{s_j \in S_j} \max_{s_i \in S_i} R_i(s)$$

($j = 1, 2,\ j \neq i$). The set F of individually rational payoffs in $R(C)$ is defined as

$$F \;=\; \{x \in R(C) \mid x_i \geq v_i,\ i = 1, 2\}.$$

With respect to the set F we make the following assumption.

Assumption 2.3 *The weak Pareto efficient frontier of F coincides with the strong Pareto efficient frontier of F.*

This assumption implies that the Pareto efficient frontier can be represented by a strictly drecreasing function.

Finally, we define the function $g_i : S \to \mathbf{R}_+$ ($i = 1, 2$) as

$$g_i(s) \;=\; \max_{\hat{s}_i \in S_i} R_i(s \backslash \hat{s}_i) \;-\; R_i(s),$$

where $s \backslash \hat{s}_i$ means that player i deviates from the pair of actions s by playing $\hat{s}_1$. The function $g_i(s)$ expresses the net gain player i can obtain by playing a best response against player j's (mixed) action $s_j \in S_j$. It is obvious that $g_i(s) \geq 0$ for all $s \in S$. Furthermore, if $s_i \in S_i$ is a best response to $s_j \in S_j$, then $g_i(s) = 0$. In what follows $g_i(s) = 0$ will used as short hand notation for saying that $s_i \in S_i$ is a best reponse against $s_j \in S_j$. A analogous interpretation has $g_i(s) > 0$ when s_i is not a best reponse. Note that $s^N \in S$ is a pair of NE actions if and only if $g_1(s^N) = g_2(s^N) = 0$. By definition, it follows that we can write $\max_{\hat{s}_i \in S_i} R_i(s \backslash \hat{s}_i) \;=\; R_i(s) + g_i(s)$. In what follows the right hand side of this equality will be frequently used to denote $\max_{\hat{s}_i \in S_i} R_i(s \backslash \hat{s}_i)$.

Behaviour Strategies.

A (proposed) contract specifies a path of actions $\pi := \{s(\tau)\}_{\tau=0}^{\infty}$, $s(\tau) \in S$, in the infinitely repeated game Γ^∞ (where $\tau = 0$ denotes the first period of the contract). Define $Q := \Pi_{\tau=0}^{\infty} S$ as the space of all feasible contracts in Γ^{∞}. The set of replies at the second stage is $\{A, R\}$, where A stands for accept and R for reject. The history of the game will be denoted by $h(t,\theta)$, $t \in \mathbb{N}$, $\theta = 1,2,3$. Formally, $h(t,1) \in H(t,1) := \Pi_{k=0}^{t-1}(Q \times \{A,R\} \times S)$, $h(t,2) \in H(t,2) := H(t,1) \times Q$ and $h(t,3) \in H(t,3) := H(t,2) \times \{A,R\}$.

A *strategy* $\sigma_i(h(t,\theta))$ for player i, $i = 1,2$, is a function which describes the action player i will take at stage θ in period t given the history $h(t,\theta) \in H(t,\theta)$. The assumptions that contracts are binding and Pareto efficient mean that if players have agreed upon the contract $\pi = \{s(\tau)\}_{\tau=0}^{\infty} \in Q$ at time t, then $\sigma_i(h(t+\tau,3)) \equiv s_i(\tau)$ for $\tau \in \mathbb{N}$.

It is assumed that both players have perfect recall. The equilibrium concept will be subgame perfectness (SPE) (see Selten [15]).

The Set of Attainable Payoffs.

The set of attainable payoffs in the bargaining model is simply the normalised discounted value of the stream of (expected) payoffs in the repeated game Γ^{∞}. Without loss of generality we will take $R(C)$ as the set of normalised payoffs in the bargaining model. This assumption is justified, because for games Γ with $R(S) \neq R(C)$ every $x \in R(C)$ is a convex combination of some $y, z \in R(S)$ iff $\delta \geq \frac{1}{2}$ (Sorin [16]). The implication of this assumption is that for any payoff vector $c^* \in R(C)$ we can find a contract C^* of infinite length that specifies some path π with normalised payoffs equal to c^*. Also for any $a \in R(C)$ there exists a pair of strategies $\sigma(h(t,\theta))$ with payoffs equal to a. In what follows a large part of the analysis will be done in the payoff space rather than in the strategy space. This means that we are able to work in the set of attainable payoffs and do not have to specify explicitly the path that is specified by the contract.

Whatever the equilibrium payoffs, it is obvious that these have to be at least the minmax value for each player. Thus, the set of equilibrium payoffs must be a subset of the set of individually rational payoffs F. Define the maximum payoff in F (F is compact) that player i, $i = 1,2$, can obtain as

$$R_i^{max} = \max \{a_i \mid (a_1, a_2) \in F\}.$$

The orthogonal projection of the set F on the x_i-axis, $i = 1,2$, is the interval $I_i = [v_i,\ R_i^{max}]$ of attainable and rational payoffs for player i. Given the interval I_i, $i = 1,2$, a function $f_j : I_i \longrightarrow I_j$, $i \neq j$, can be defined that describes the maximum payoff in F for player j given payoff $x_i \in I_i$ for player i. Formally, $f_1 : I_2 \longrightarrow I_1$ and $f_2 : I_1 \longrightarrow I_2$ are defined as

$$f_1(x_2) \;=\; \max\,\{a_1 \mid (a_1, x_2) \in F\} \quad \text{and} \quad f_2(x_1) \;=\; \max\,\{a_2 \mid (x_1, a_2) \in F\}.$$

The function $f_j(x_i)$ is a continuous and concave function on I_i, because F is compact and convex. Assumption 2.3 implies that the function f_j is single peaked. We define R_i^* as the argument of f_j for which this function attains its maximum, that is $R_j^{max} = f_j(R_i^*)$. Note that (R_i^*, R_j^{max}) is one of the two endpoints of the curve of Pareto efficient payoffs of the set F.

The bargaining procedure of this paper allows for renegotiation of inefficient contracts. Any subgame in which an inefficient contract is agreed upon can be reformulated as an alternating offer model [14] in which the cake is represented by the set F and the status quo payoffs are determined by the inefficient contract. From (Rubinstein [14]) it follows that this subgame has a unique SPE in which players reach an immediate agreement which is also Pareto efficient. In other words, each inefficient contract will be immediately replaced by some new and efficient contract.[2] Therefore, we make the following simplifying assumption.

Assumption 2.4 *Only Pareto efficient contracts are permitted.*

Finally, the set $F^+ \subseteq F$ is defined as

$$F^+ \;:=\; \{x \in F \mid x \geq (R_1^*, R_2^*)\ \}.$$

The set F^+ is non-empty and compact, because F is compact and $(R_1^*, R_2^*) \in F$. The following example illustrates the several definitions.

Example 2.1 (Battle-of-the-sexes)

	L_2	R_2
L_1	(1,3)	(0,0)
R_1	(0,0)	(3,1)

This game has the property that $R(S) \neq R(C)$. For $\delta \geq \frac{1}{2}$ the payoff space is

$R(C) = convexhull\{(0,0),(1,3),(3,1)\}$. The minmax values $v_1 = v_2 = \frac{3}{4}$, the set of individually rational payoffs $F = \{x \in R(C) \mid x \geq (\frac{3}{4},\frac{3}{4})\}$, $I_1 = I_2 = [\frac{3}{4},3]$ and $R_1^{max} = R_2^{max} = 3$. The function $f_2(x_1)$ is

$$f_2(x_1) = \begin{cases} 3x_1, & x_1 \in [\frac{3}{4},1), \\ 4 - x_1, & x_1 \in [1,3] \end{cases}$$

and attains it maximum in $R_1^* = 1$. The decreasing part of $f_2(x_1)$ describes the Pareto efficient frontier of F. Furthermore, $F^+ = \{x \in F \mid x \geq (1,1)\}$. Thus, $F^+ \neq F$ for this example.

3 Properties of the Set of Equilibrium Payoffs

In this section some properties of the set of equilibrium payoffs are derived. First, it will be shown that the set of SPE payoffs is non-empty by showing that at least one stationary SPE exists.[3] Stationary SPE's are closely related to the unique SPE of the alternating offer model if in the latter model the cake is represented by the set F and one of the NE's is taken as the exogenously given status quo (or threat) point. In the second part of this section it will be shown that all the limit SPE payoffs, as δ goes to 1, belong to the subset F^+. Thus, the limit set of SPE payoffs has to be contained in the set F^+.

Proposition 3.1 *There exists at least one stationary SPE.*

Proof

The conditions on Γ are sufficient to ensure that at least one NE exists. Choose a NE of Γ and denote $s^N \in S$ as the corresponding NE actions. Define the function $p : F \to F$ as

$$p_2(y) = \max\{R_2^*, (1-\delta)R_2(s^N) + \delta y_2\}, \quad p_1(y) = f_1(p_2(y)),$$

and the function $q : F \to F$ as

$$q_1(x) = \max\{R_1^*, (1-\delta)R_1(s^N) + \delta x_1\}, \quad q_2(x) = f_2(q_1(x)).$$

Brouwer's fixed point theorem can be applied to prove that $p \times q : F \times F \to F \times F$ has a fixed point $(x, y) = (p(y), q(x))$. Consider the following strategies

Player 1 proposes $x \in F$ and player 2 accepts if $x_2 \geq (1-\delta)R_2(s^N) + \delta y_2$.
Player 2 proposes $y \in F$ and player 1 accepts $y_1 \geq (1-\delta)R_1(s^N) + \delta x_1$.
As long as both players do not agree the disagreement actions are s^N.

These stationary strategies are SPE if and only if (x, y) is a fixed point of the function $p \times q$. To see this, note that the gain $g_i(s^N) = 0$, $i = 1, 2$ so that none of the players has an incentive to deviate at the third stage. Given y is the SPE outcome at the next period, player 2 can guarantee himself $(1-\delta)R_2(s^N) + \delta y_2$ and this player will not accept less than this value. Therefore, in case $R_2^* < (1-\delta)R_2(s^N) + \delta y_2$, player 1's best proposal is x and in the other case, $R_2^* \geq (1-\delta)R_2(s^N) + \delta y_2$, player 1's best proposal is $x = (R_1^{max}, R_2^*)$. Hence, x is an SPE outcome given y is an SPE outcome one period later. Similarly, y is an SPE outcome when x is an SPE outcome. □

Corollary 3.1 *If $s^N \in S$ is not Pareto efficient and players follow the stationary SPE strategies above, then (i) it is advantagous to be the first proposer, (ii) both players reach an immediate agreement, (iii) the contracts are Pareto efficient and (iv) x and y converge to the symmetric Nash bargaining solution with threat point $R(s^N)$ as δ goes to 1.*

In the next sections much attention is paid to the limit set of SPE payoffs as δ goes to 1. Here we will derive a simple convergence result that says nothing more than that player i can guarantee himself at least R_i^* as δ goes to 1. Hence, in the limit, every SPE payoff for player i has to be greater than or equal to this level. This implies that every limit SPE payoff $a \in F$ has to satisfy the following condition: $a \geq (R_1^*, R_2^*)$, or equivalently $a \in F^+$. The reader is warned that this proposition does not say that the limit set of SPE payoffs equals the set F^+. Examples 6.2 and 6.3 illustrate this point.

Proposition 3.2 *For any period t ($t \in \mathbb{N}$): The set of SPE payoffs converges to a subset of F^+ as δ goes to 1.*

Proof.

If $F^+ = F$ the statement is trivial. Therefore, assume that $F^+ \subset F$. Suppose $\sigma(h(t, \theta))$ is a pair of strategies such that player 1's payoff is strictly less than R_1^* at time t (t even). Consider the following deviation for player 1. Player 1 proposes the contract C with payoffs $c = (R_1^*, R_2^{max})$ at the first stage of period t. Whatever, the pair of strategies $\sigma(h(t,\theta))$ is, player 2's continuation payoff of following this pair of strategies will be at most R_2^{max}. Therefore, player 2 will accept the proposed contract C. Hence, this deviation is profitable for player 1 and $\sigma(h(t,\theta))$ can not be a pair of SPE strategies.

Given that player 1's SPE payoff at any time t (t even) cannot be less than R_1^* it follows that player 1 can secure himself a payoff of $(1-\delta)v_1 + \delta R_1^*$ at the third stage of period $t-1$. Therefore, at the second stage of period $t-1$, player 1 will certainly reject any contract that yields a payoff lower than this value. When δ goes to 1 it follows that player 1's security level goes to R_1^*. At the third stage of period $t-2$ player 1 can again secure himself of a payoff that is at least v_1 in $t-2$ and a continuation payoff of at least $(1-\delta)v_1 + \delta R_1^*$. Therefore, player 1's security level at the third stage of period $t-2$ is at least $(1-\delta^2)v_1 + \delta^2 R_1^*$. Again, this value converges to R_1^* as δ goes to 1. Hence, player 1's security level at any stage and in any period converges to R_1^* as δ goes to 1.

Similarly, player 2's security level at any stage and in any period converges to R_2^* as δ goes to 1. This means that every element $a \in F$ that belongs to the limit set of SPE payoffs has to satisfy $a \geq (R_1^*, R_2^*)$. □

4 "Immediate Agreement" Strategies

The strategies introduced and analysed in this section are called "immediate agreement" strategies, because if players follow this type of strategies then they immediately reach agreement in every subgame. The stationary strategies of proposition 3.1 are a special case of the type of strategies in this section. When players follow the stationary SPE strategies of proposition 3.1 the payoffs of the proposed contract depend heavily upon the disagreement outcome when players would end up in perpetual disagreement. In particular, as δ goes to 1, the stationary SPE outcome converges to the (symmetric) Nash bargaining solution with the perpetual disagreement outcome as the threat vector. The higher player j's disagreement payoff ($j = 1,2$) in all periods t where player i ($i = 1,2,\ i \neq j$) proposes, the higher player j's SPE payoff will be. Similarly, the lower player i's disagreement payoff in all periods where player j proposes, the higher player j's SPE payoff will be. This means that if other disagreement actions can be supported in an SPE, then new SPE's are obtained. Using stationary strategies, as in the previous section, is not enough to obtain new SPE's, because all stationary SPE strategies must satisfy that the disagreement actions are best responses against each other in order to prevent unilateral deviation. In other words, the disagreement actions have to be NE actions in case stationary SPE strategies are constructed.

The question addressed in this section is how can more general strategies be constructed that yield a higher SPE payoff to player j ($j = 1, 2$) but have different disagreement actions than NE actions. In other words, these strategies have to satisfy *i)* player j's disagreement payoff in periods where the other player proposes is at least as high as the "best" NE payoff for this player, *ii)* player i's disagreement payoff ($i = 1, 2$, $i \neq j$) in all periods where player j proposes is at most this player's "worst" NE payoff, *iii)* both players reach immediate agreement in every period and *iv)* no player has an incentive to deviate from playing these strategies. Without loss of generality, player i is player 1 and player j is player 2.

The type of strategies in this section, introduced by Bush and When [2], are such that in all odd periods player 1 is rewarded with some good contract offered by player 2 when player 1 has used the prescribed disagreement actions one period before and is punished with some bad contract offered by player 2 after a deviation by player 1. Thus, the contract offered by player 2 depends upon the actual action player 1 has used one period before. One could call these strategies "compensation" strategies, because player 1 is compensated by the good contract for the loss of not playing a best response against the disagreement action of player 2 one period earlier.

Before the strategies are described formally, we first define the state dependent contract player 2 will offer player 1. Let $c \in \mathbf{R}_+$ denote the compensation player 2 offers player 1 at time $t+1$ (t even) and define the state dependent contract $y : S \to \mathbf{R}$ (at time $t + 1$) as follows

$$y_1(s(t)) = \begin{cases} \alpha + c, & \text{if } s_1(t) = s_1^e, \\ \alpha, & \text{otherwise,} \end{cases}$$

$$y_2(s(t)) = f_2(y_1(s(t))),$$

where $s(t)$ denotes the actual pair of actions used at time t, α player 1's payoff independent of the compensation and $s_1^e \in S_1$ the action player 1 is supposed to play at time t.

Let x^*, y^* and s^N be a stationary SPE strategy as formulated in proposition 3.1 and choose s^N such that the stationary SPE strategy is the stationary SPE strategy that yields the highest payoff to player 2. The "immediate agreement" strategies, introduced by Bush and When [2], are defined as follows.

t even	Stage 1	Player 1 proposes a contract with payoff $x_2 = \max\{R_2^*, (1-\delta)R_2(s^e) + \delta y_2(s^e)\}$ and $x_1 = f_1(x_2)$.
	Stage 2	Player 2 accepts x if $x_2 \geq (1-\delta)R_2(s^e) + \delta y_2(s^e)$.
	Stage 3	If players did not agree before, then s^e are the disagreement actions with payoff $R(s^e)$.
$t+1$	Stage 1	Player 2 proposes a contract with payoff $y_1(s^e) = \max\{R_1^*, (1-\delta)R_1(s^o) + \delta x_1\} + c$ and $y_2(s^e) = f_2(y_1(s^e))$,
	Stage 2	Player 1 accepts y if $y_1 \geq (1-\delta)R_1(s^o) + \delta x_1$.
	Stage 3	If players did not agree before, then s^o are the disagreement actions with payoff $R(s^o)$.

If player 2 deviates, then both players immediately follow the stationary SPE strategy with contracts x^* and y^*.

The notation is chosen in such a way that $s^e \in S$ and $s^o \in S$ denote the disagreement actions at the even periods and at the odd periods respectively. The next proposition states the necessary and sufficient conditions that have to be satisfied for these strategies in order to be SPE strategies.

Proposition 4.1

The "immediate agreement" strategies are SPE strategies if and only if

i) $0 \leq (1-\delta)g_2(s^o) \leq \delta(x_2 - x_2^*)$ *and* $g_1(s^o) = 0$,

ii) $0 \leq (1-\delta)g_2(s^e) \leq \delta(y_2 - y_2^*)$ *and* $c \geq \frac{1-\delta}{\delta} g_1(s^e) \geq 0$,

iii) $c \leq \max\{(1-\delta)R_1(s^N) + \delta x_1^*, R_1^*\} - \max\{(1-\delta)R_1(s^o) + \delta x_1, R_1^*\}$,

Proof.

($\Rightarrow$) Let $t \in \mathbb{N}$ be even and consider $t+2$. The strategies are SPE, thus the contract

x proposed by player 1 will be accepted by player 2. Therefore, x is the SPE continuation payoff. Because the strategies are SPE the following two incentive constraints at the third stage of period $t+1$ have to hold

$$(1-\delta)[R_1(s^o)+g_1(s^o)]+\delta x_1 \leq (1-\delta)R_1(s^o)+\delta x_1 \tag{1}$$

and

$$(1-\delta)[R_2(s^o)+g_2(s^o)]+\delta x_2^* \leq (1-\delta)R_2(s^o)+\delta x_2. \tag{2}$$

(1) is equivalent to $g_1(s^o) \leq 0$, which has to be the =-sign because $g_1(s) \geq 0$ by definition, and (2) can be rewritten as $(1-\delta)g_2(s^o) \leq \delta(x_2 - x_2^*)$. Given $g_2(s^o) \geq 0$ (by definition) it has to hold that $x_2 \geq x_2^*$.

Player 1 will accept $y_1(s(t))$ if $y_1(s(t)) \geq (1-\delta)R_1(s^o)+\delta x_1$. Hence, $c \geq 0$ in order to have this constraint to hold for all $s(t) \in S$. The incentive constraint for player 2, in case $s_1(t) = s_1^e$, for the offer $y(s^e)$ is

$$\begin{aligned} y_2(s^e) &= f_2\big(\max\{R_1^*, (1-\delta)R_1(s^o)+\delta x_1\}+c\big) \\ &\geq f_2(\max\{R_1^*, (1-\delta)R_1(s^N)+\delta x_1^*\}) = y_2^*. \end{aligned}$$

This completes the proof for period $t+1$.

At the third stage of period t the following two incentive constraints have to hold

$$(1-\delta)[R_1(s^e)+g_1(s^e)]+\delta\beta \leq (1-\delta)R_1(s^e)+\delta[c+\beta], \tag{3}$$

where $\beta = \max\{R_1^*, (1-\delta)R_1(s^o)+\delta x_1\}$, and

$$(1-\delta)[R_2(s^e)+g_2(s^e)]+\delta y_2^* \leq (1-\delta)R_2(s^e)+\delta y_2(s^e). \tag{4}$$

(3) is equivalent to $\delta c \geq (1-\delta)g_1(s^e) \geq 0$ and (4) can be rewritten as $(1-\delta)g_2(s^e) \leq \delta(y_2 - y_2^*)$. Given $g_2(s^e) \geq 0$ it must be that $y_2 \geq y_2^*$.

Finally, $y_2(s^e) = f_2(y_1(s^e)) \geq f_2(y_1^*) = y_2^*$ is equivalent to $y_1(s^e) \leq y_1^*$. Rewriting of this last inequality yields

$$\max\{(1-\delta)R_1(s^N)+\delta x_1^*, R_1^*\} - \max\{(1-\delta)R_1(s^o)+\delta x_1, R_1^*\} \geq c.$$

This completes the first part of the proof.

($\Leftarrow$) The arguments of the proof of proposition 3.1 can be applied to prove that, given $R(s^e)$ and $R(s^o)$, x and $y(s^e)$ exist. The rest follows trivially from the first part of the proof because all incentive constraints are equivalent to the conditions stated in the proposition. □

Corollary 4.1 *Suppose $x = x^*$ and $y(s^e) = y^*$.*
The "immediate agreement" strategies are SPE strategies if and only if s^e and s^o are NE actions.

Proof.
First, condition *iii)* implies that $g_2(s^o) = g_2(s^e) = 0$ and $g_1(s^0) = 0$. Secondly, $y_1^* = y_1(s^e)$ combined with condition *iv)* implies that the equilibrium compensation c equals 0. Therefore, $g_1(s^e) = 0$. This means that s^e and s^o belong to the set of NE of Γ. □

Given these necessary and sufficient conditions an important question is what the procedure is to find the "immediate agreement" SPE strategies that yield player 2's highest SPE payoff given this type of strategies. Thus, the problem is

$$\max_{s^e, s^o \in S,\ c \in \mathbb{R}_+} x_2, \qquad \text{s.t. conditions } i)\text{-}iii) \text{ of proposition 4.1.}$$

Note that this optimisation problem also yields player 1's lowest SPE payoff given this type of strategies, because the SPE contract x is Pareto efficient (that is $x_1 = f_1(x_2)$) and $f_2(\max x_2) = \min f_1(x_2) = \min x_1$ (f_1 is strictly decreasing for $x_2 \geq R_2^*$). The next proposition generalises the result of Bush and When [2] to the class of games considered in this paper.

Proposition 4.2 *Suppose players follow "immediate agreement" SPE strategies. Sufficient conditions to find the SPE strategy that yields player 2's maximum payoff, denoted by $\hat{s}^e, \hat{s}^o, \hat{c}$, are*

i) $\hat{s}^o = m^1$, *(player 1 is minmaxed)*

ii) $\hat{c} = \frac{1-\delta}{\delta} g_1(\hat{s}^e)$, *(minimal compensation)*

iii) $\hat{s}^e = \arg\max_{s^e \in S}$
$\max\{R_2^*, (1-\delta)R_2(s^e) + \delta f_2\big(\max\{R_1^*, (1-\delta)[v_1 + \delta^{-1} g_1(s^e)] + \delta x_1\}\big)\}$.

Proof.
From standard bargaining theory [14] it follows that, given s^e, s^o and c, the payoff x_1 is uniquely determined. Define $a = (1-\delta)R_2(s^e)$ and $b = (1-\delta)R_1(s^o) + c$, then $x_1(a, b)$ is a fixed point that satisfies

$$x_1(a,b) = l(x_1(a,b);a,b) := f_1(\max\{R_2^*, a + \delta f_2(\max\{R_2^*, b + \delta x_1\})\}).$$

Consider $a' < a$ and $b' > b$, then it follows that for every $z \in I_1$ the following relation holds $l(z;a,b) \leq l(z;a',b')$. Hence, $x_1(a,b) \leq x_1(a',b')$.

Minimising the value of b and taking into account the conditions of proposition 4.1 yields $c = \frac{1-\delta}{\delta} g_1(s^e)$ and $m^1 = \arg\min_{s^o \in S} R_1(s^o)$, s.t. $g_1(s^o) = 0$. Substitution of these results into b implies that both a and b are functions of s^e. This implies that it is not possible to find $\hat{s}^e$ as the solution of a simple optimisation problem, but one has to solve the complex optimisation problem stated as condition *iii)*. Thus, these three conditions are sufficient in order to find $\max x_2$, because $\max x_2 = f_2(\min x_1)$. □

Remark 4.1

The function $g_1(s)$ need not be continuous. This implies that a solution of condition *iii)* may fail to exist. In that case it is possible to find player 2's supremum SPE payoff and one has to approximate the argument for which this supremum SPE payoff is attained.

Remark 4.2

The complex optimisation problem of condition *iii)* reflects the trade off between a higher value of $R_2(s^e)$ and a lower value of compensation $\frac{1-\delta}{\delta} g_1(s^e)$. To illustrate that the Pareto boundary of the set F is also important in this trade off suppose that the function f_2 is continuously differentiable and $F = F^+$. The first order Taylor expansion is equal to

$$\hat{s}^e = \arg\max_{s^e \in S} R_2(s^e) + f_2'(.)g_1(s^e),$$

which reflects that the Pareto boundary is important in this trade off.
(Note that $f_2'(.)$ is negative.)

Remark 4.3

There is a subclass of games for the bargaining model in this paper that also satisfy the assumptions Bush and Wen [2] make. The games in this subclass satisfy $f_2(x_1) = 1 - x_1$ and $v_1 = v_2 = 0$ (see also examples 6.2 and 6.3). For this subclass of games it easily follows that

$$x_2 = \frac{\delta[1 - R_1(s^o)] + [R_2(s^e) - g_1(s^e)]}{1+\delta}.$$

Condition *iii)* reduces to the following optimisation problem

$$\max_{s^e \in S} [R_2(s^e) - g_1(s^e)].$$

5 "One Sided Offer" Strategies

The strategies introduced and analysed in this section are called "one sided offer" strategies, because if both players follow these strategies then it is as if only one player makes proposals (that are accepted) while the other player does not make any proposal (because this player's proposals are "unacceptable" and therefore rejected). This section is organised as follows. In the first subsection the "one sided offer" strategies are defined and then necessary and sufficient conditions are derived under which these strategies are SPE strategies. One of these conditions will be restated to obtain a simple graphical interpretation of this condition which is also usefull in computing the SPE strategy that yields a player highest SPE payoff given this type of strategies.

The second subsection is concerned with the relation between weakly renegotiation proof (WRP) payoffs for standard repeated games (Farrell and Maskin [7]) and SPE payoffs derived in the first subsection. It will be shown that WRP payoffs can be sustained as SPE payoffs in the bargaining game when players use "one sided offer" strategies provided the WRP payoffs satisfy one additional condition. This additional condition is automatically satisfied when a WRP payoff is Pareto efficient.

5.1 Equilibrium Conditions

Before the "one sided offer" strategies are formally defined some additional notation is introduced. Without loss of generality we assume that player 2 makes the "acceptable" proposals and player 1 the "unacceptable" proposals.

As mentioned in section 2 each $a \in F$ can be approximated arbitrarily close by at most two vectors $a^*, a^{**} \in R(S)$ such that the continuation payoff at any period is also arbitrarily close to a. Let $s^*, s^{**} \in S$ denote the corresponding actions to a^*, a^{**} respectively, and let $< s(t) >$, $s(t) \in \{s^*, s^{**}\}$, be the infinite sequence of actions that approximates a. As in the previous section, any deviation of player 2 is punished by the stationary SPE strategy with $R(s^N)$ as the NE payoffs in Γ that are "best" for player 2 and x^* and y^* the SPE payoffs made by player 1 and player 2 respectively.

The "one sided offer" strategies are similar to the strategies used in the repeated games literature in the sense that a normal phase and a punishment phase are constructed. Denote the length of player 1's punishment phase as T, T is assumed to be even, and let τ denote the number of periods player 1 is already punished after this player's last (unilateral) deviation. The following convention is adopted: If player 1 deviates from playing the normal phase then the counting starts with $\tau = 2$ in the punishment phase at the first odd period *after* this deviation, and if player 1 deviates during the punishment phase counting starts with $\tau = 0$ from the first odd period *after* this deviation. (Thus, counting does never start at even periods. After a deviation from the normal phase, it is as if the punishment phase had already started with $\tau = 0$ at the last odd period before this deviation occurred in the past.) Finally, given $s^p \in S$, player 1's continuation payoff after being punished τ periods in the punishment phase is defined as

$$p_1(\tau) = (1 - \delta^{T-\tau})R_1(s^p) + \delta^{T-\tau}a_1.$$

The "one sided offer" strategies, given by $< s(t) >$, are defined as follows.

normal phase

t even	Stage 1	Player 1 proposes a contract with payoff (R_1^{max}, R_2^*).
	Stage 2	Player 2 accepts every contract that yields at least $(1-\delta)R_2(s^p) + \delta f_2(p_1(2))$.
	Stage 3	If players did not agree before, then $s(t)$ are the disagreement actions with payoff $R(s(t))$.
$t+1$	Stage 1	Player 2 proposes a contract with payoff $y(t) = (a_1, f_2(a_1))$.
	Stage 2	Player 1 accepts every contract that yields at least a_1.
	Stage 3	If players did not agree before, then $s(t+1)$ are the disagreement actions with payoff $R(s(t+1))$.

punishment phase

t even	Stage 1	Player 1 proposes a contract with payoff (R_1^{max}, R_2^*).
	Stage 2	Player 2 accepts every contract that yields at least $(1-\delta)R_2(s^p) + \delta f_2(p_1(0))$.
	Stage 3	If players did not agree before, then s^p are the disagreement actions with payoff $R(s^p)$.
$t+1$	Stage 1	Player 2 proposes a contract with payoff $y^p(\tau) = (p_1(\tau), f_2(p_1(\tau)))$.
	Stage 2	Player 1 accepts every contract that yields at least $p_1(\tau)$.
	Stage 3	If players did not agree before, then s^p are the disagreement actions with payoff $R(s^p)$.

Return to the normal phase when $\tau = T$.
Restart the punishment phase after any deviation of player 1.

Whenever player 2 deviates, then both players immediately follow the stationary SPE strategy with contracts x^* and y^*.

The notation is chosen in such a way that $s^p \in S$ denotes the disagreement actions in the punishment phase. The next proposition states the necessary and sufficient conditions that have to be satisfied in order for these strategies to be SPE strategies.

Proposition 5.1 *Let $a \in F^+$. If the following conditions hold*

i) $\Sigma_{k=1}^{T}\delta^k \geq [R_1^{max} - a_1]/[a_1 - R_1(s^p)]$,

ii) $a_1 \geq (1-\delta)R_1^{max} + \delta[R_1(s^p) + g_1(s^p)]$,

iii) $(1-\delta)R_2(s^p) + \delta f_2(p_1(2)) \geq f_2(a_1)$ *and*

$(1-\delta)R_2(s^p) + \delta f_2(p_1(0)) \geq f_2(p_1(1))$

then the "one sided offer" strategies are SPE strategies for sufficiently large $\delta < 1$. The strict version of ii) is necessary if there does not exists an $s \in S$ such that simultaneously $g_1(s) = 0$ and $R(s)$ is Pareto efficient.

Proof.
(necessary conditions) In order for the "one sided offer" strategies to be SPE strategies the following incentive constraints in the normal phase have to hold.

Consider player 1 first. First of all, player 1 should not have an incentive to deviate from $s(t)$ and $s(t+1)$. This implies that the following two incentive constraint have to be satisfied

$$(1-\delta)R_1^{max} + \delta p_1(2) \leq a_1 \tag{5}$$

and

$$(1-\delta)[R_1^{max} + \delta R_1(s^p)] + \delta^2 p_1(2) \leq a_1.$$

(5) implies that $p_1(2) \leq a_1$ (because $a_1 \leq R_1^{max}$). This means that the second inequality holds whenever (5) holds. Therefore, this second inequaltiy is disregarded in what follows.

Player 1 will not deviate from proposing the contract with payoff (R_1^{max}, R_2^*) if there does not exists a contract with payoff $c \in F$ such that $c_1 > a_1$ and $c_2 \geq (1-\delta)R_2(s^p) + \delta f_2(p_1(2))$. This means that the following incentive constraint has to be satisfied

$$(1-\delta)R_2(s^p) + \delta f_2(p_1(2)) \geq f_2(a_1). \tag{6}$$

Player 2 does not have an incentive to deviate from the normal phase if $f_2(a_1) \geq y_2^*$, $(1-\delta)R_2^{max} + \delta x_2^* \leq (1-\delta^2)a_2 + \delta^2 f_2(a_1)$ and $(1-\delta)R_2^{max} + \delta y_2^* \leq (1-\delta)a_2 + \delta f_2(a_1)$. These constraints are certainly satisfied if the first inequality is strict and for sufficiently large $\delta < 1$.

Secondly, the incentive constraints for the punishment phase are derived. Consider player 1 first. Player 1 should not have an incentive to deviate from s^p. This implies that the following two incentive constraints have to be satisfied

$$(1-\delta)[R_1(s^p) + g_1(s^p)] + \delta p_1(0) \leq p_1(\tau), \tag{7}$$

in case τ odd and $\tau \geq 1$, and

$$(1-\delta)[R_1(s^p) + g_1(s^p) + \delta R_1(s^p)] + \delta^2 p_1(0) \leq p_1(\tau),$$

in case τ even and $\tau \geq 2$. As in the normal phase, this last inequality holds whenever (7) holds and can therefore be disregarded. Furthermore, player 1 must not have an incentive to deviate forever (by rejecting and playing a best reponse to s_2^p forever), that is

$$R_1(s^p) + g_1(s^p) \leq p_1(\tau). \tag{8}$$

Equations (7) and (8) can only hold iff either $a_1 > R_1(s^p) + g_1(s^p)$ and for sufficiently large $\delta < 1$ (note $\lim_{\delta \to 1} p_1(\tau) = a_1$, provided T is finite), or $a_1 = R_1(s^p) + g_1(s^p)$ and $g_1(s^p) = 0$ (if not $g_1(s^p) = 0$, then $p_1(\tau) < R_1(s^p) + g_1(s^p)$ for all $\delta < 1$ violating (7)).

Player 1 will not deviate from proposing (R_1^{max}, R_2^*) at the τ-th period of the punishment phase if there does not exist a contract with payoff c such that $c_1 > p_1(\tau)$ and $c_2 \geq (1-\delta)R_2(s^p) + \delta f_2(p_1(0))$. If τ is odd, then this constraint can be rewritten as

$$(1-\delta)R_2(s^p) + \delta f_2(p_1(0)) \geq f_2(p_1(\tau)), \tag{9}$$

provided $R_1(s^p) < a_1$.

Substitution of $p_1(2)$ into (5) and rewriting yields

$$\Sigma_{k=1}^{T-1}\delta^k = \delta\frac{1-\delta^{T-2}}{1-\delta} \geq \frac{R_1^{max} - a_1}{a_1 - R_1(s^p)}.$$

Finally, $a_1 \geq p_1(\tau+1) \geq p_1(\tau) \geq p_1(1) \geq p_1(0)$ implies

$$f_2(p_1(1)) \geq f_2(p_1(\tau)) \geq f_2(p_1(\tau+1))$$

and

$$(1-\delta)R_2(s^p) + \delta f_2(p_1(2)) \leq (1-\delta)R_2(s^p) + \delta f_2(p_1(0)).$$

Therefore, both equations (6) and (9) for $\tau = 1$ have to hold.

Finally, if $g_1(s^p) = 0$ and $a = R(s^p)$, then $p_1(\tau) = a_1$ and equations (6) and (9) both reduce to

$$(1-\delta)a_2 + \delta f_2(a_1) \geq f_2(a_1),$$

which can only hold when $a_2 \geq f_2(a_1)$. Hence, $a = R(s^p)$ has to be Pareto efficient in this case.

Player 2 does not have an incentive to deviate in the punishment phase provided player 2 does not have an incentive to deviate in the normal phase, because the punishment phase yields this player a higher payoff than the normal phase. □

The way in which the third condition of proposition 5.1 is formulated has no obvious interpretation. However, this condition can be restated if a first order Taylor expansion is used to approximate $f_2(p_1(\tau))$ $(\tau = 0, 1, 2)$. This approximation will be in the point a_1, because, by definition, $p_1(\tau)$ $(\tau = 0, 1, ..., T)$ converges to a_1 as δ goes to 1. Because emphasis will be on limit SPE payoffs as δ goes to 1 this point is a natural candidate. Define $f_2'(a_1)$ as the largest subdifferentiable in the point $a_1 \in [R_1^*, R_1^{max}]$.[4] The first order Taylor expansion of $f_2(p_1(\tau))$ $(\tau = 0, 1, ..., T)$ is given by

$$f_2(p_1(\tau)) \approx f_2(a_1) + f_2'(a_1)[p_1(\tau) - a_1].$$

The concavity of the function f_2 implies that the approximation in the point x_1 yields a higher value than the true value of f_2 in this point. In order to reformulate condition *iii)* it is necessary that the approximation is at most $f_2(p_1(\tau))$ when $\tau = 0, 2$. This can be achieved for sufficiently small $\epsilon > 0$ and sufficiently large $\delta < 1$, because the line $f_2(a_1) + [f_2'(a_1) + \epsilon](p_1(\tau) - a_1)$ intersects the curve of f_2 twice, once in some point $x_1(\epsilon) < a_1$ and once in the point a_1. For values in between these points the function f_2 lies above this line. Hence, for sufficiently large $\delta < 1$ all points $p_1(\tau)$ $(\tau = 0, 1, ..., T)$ are larger than $x_1(\epsilon)$ and $f_2(p_1(\tau)) > f_2(a_1) + [f_2'(a_1) + \epsilon](p_1(\tau) - a_1))$. In what follows we disregard the $\epsilon > 0$ and we leave it to the reader to verify that whenever the conditions of the following proposition hold, then there exists a sufficiently small $\epsilon > 0$ such that the statement also holds in the limit as δ goes to 1 given this $\epsilon > 0$. Note that if the function f_2 is piecewise linear then this problem does not arise and ϵ can be taken equal to 0.

Applying the first order Taylor expansion implies that condition *iii)* of proposition 5.1 can be restated as in the following lemma for sufficiently large $\delta < 1$.

Lemma 5.1 *For sufficiently large $\delta < 1$ condition iii) of proposition 5.1 is equivalent to*

$$R_2(s^p) > f_2(a_1) + f_2'(a_1)[a_1 - R_1(s^p)].$$

The strict version is necessary if either f_2 is not piecewise linear or $a = R(s^p)$ and $g_1(s^p) = 0$ but a is not Pareto efficient.

Proof.
Substitution of the first order Taylor approximation and the definition of $p_1(2)$ into the first inequality of condition *iii)* of proposition 5.1 yields

$$(1-\delta)[f_2(a_1) - R_2(s^p)] + f_2'(a_1)\delta(1-\delta^{T-2})[a_1 - R_1(s^p)] \leq 0,$$

or, equivalently,

$$\begin{aligned} R_2(s^p) &\geq f_2(a_1) + f_2'(a_1)\tfrac{\delta(1-\delta^{T-2})}{(1-\delta)}[a_1 - R_1(s^p)] \\ &= f_2(a_1) + f_2'(a_1)\Sigma_{k=1}^{T-2}\delta^k[a_1 - R_1(s^p)]. \end{aligned}$$

Hence, as δ goes to 1,

$$R_2(s^p) \geq f_2(a_1) + (T-2)f_2'(a_1)[a_1 - R_1(s^p)]. \tag{10}$$

Similarly, the second inequality of condition *iii)* of proposition 5.1 can be rewritten as

$$(1-\delta)[f_2(a_1) - R_2(s^p)] - (1-\delta)[1-(1+\delta)\delta^{T-1}]f_2'(a_1)[a_1 - R_1(s^p)] \leq 0.$$

Dividing both sides by $(1-\delta)$ and rewriting yields

$$\begin{aligned} R_2(s^p) &\geq f_2(a_1) - [1 - \delta^{T-1}(1+\delta)]f_2'(a_1)[a_1 - R_1(s^p)] \\ &\to f_2(a_1) + f_2'(a_1)[a_1 - R_1(s^p)], \end{aligned}$$

as $\delta \to 1$. If this last inequality holds, then also equation (10) holds (because $f_2'(a_1) < 0$).

The strict inequality follows from the fact that we should take a sufficiently small $\epsilon > 0$ in case the function is not piecewise linear as argued informally. □

The condition $R_2(s^p) > f_2(a_1) + f_2'(a_1)[a_1 - R_1(s^p)]$ has a simple graphical interpretation. The Taylor approximation is a line l tangent to the curve of f_2 at the point $(a_1, f_2(a_1))$. The reflection of the line l, with the horizontal line through the point $(a_1, f_2(a_1))$ as the mirror line, is denoted as the line l'. The condition of the previous lemma states that the point $R(s^p)$ has to lie above the line l' in order for a to be an SPE of the bargaining game. Denote $p(\tau) = (p_1(\tau), f_2(p_1(\tau)))$, then this means that the convergence of $(1-\delta)R(s^p) + \delta p(\tau)$ towards the point $(a_1, f_2(a_1))$ (with approximated direction $\frac{f_2(a_1) - R_2(s^p)}{a_1 - R_1(s^p)}$) has to be faster than the convergence of $p(\tau)$ towards $(a_1, f_2(a_1))$ (with approximated direction $f_2'(a_1)$).

The last result of this subsection is the procedure to find the SPE strategies that approximate player 2's highest SPE payoff given these strategies.

Proposition 5.2 *Define $a_1(s) = R_1(s) + g_1(s)$. Player 2's best "one sided offer" SPE strategy can be found by solving the following optimisation problem*

$$s^* = \arg\min_{s \in S} a_1(s), \quad s.t. \quad R_2(s) \geq f_2(a_1(s)) + f_2'(a_1(s))\; g_1(s).$$

Proof.
Condition *(ii)* of proposition 5.1 states that $a_1 > R_1(s) + g_1(s)$. However, as δ goes to 1 every a_1 can be chosen arbitrarily close to $R_1(s) + g_1(s)$ as long as

$$T - 1 = \lim_{\delta \to 1} \Sigma_{k=1}^{T-1} \delta^k > \frac{R_1^{max} - a_1}{a_1 - R_1(s)}.$$

Without loss of generality we can take $a_1(s)$ as the approximated value. The function f_2 is strictly decreasing on $[R_1^*, R_1^{max}]$ and therefore $\max_{s \in S} f_2(a_1(s))$ is the same as $\min_{s \in S} a_1(s)$. Obviously, the optimisation problem has to satisfy also the condition of lemma 5.1. Because $g_1(s) = a_1(s) - R_1(s)$ by definition and $R_2(s)$ arbitrarily close to $f_2(a_1(s)) + f_2'(a_1(s))[a_1(s) - R_1(s)]$ the $\geq$-sign can be written without loss of generality. □

Remark 5.1
The function $g_1(s)$ need not be continuous. In that case $\inf_{s \in S} R_1(s) + g_1(s)$ has to be approximated arbitrarily close and s^* can be taken as the argument of this approximation.

5.2 Relation to Weakly Renegotiation Proof Equilibria

In this subsection the relation between weakly renegotiation proof (WRP) payoffs for standard repeated games (Farrell and Maskin [7]) and "one sided offer" SPE payoffs is investigated. Sufficient conditions for the payoff $a \in F$ to be a WRP payoff is that there exists $s^i \in S$ $(i = 1, 2)$ such that

1. $a_i > R_i(s^i) + g_i(s^i)$,
2. $a_j \geq R_j(s^i)$,

with $(j = 1, 2,\ j \neq i)$. The strict inequality of the first condition is necessary if $g_i(s_i) \neq 0$ and is allowed to be an =-sign when $g_i(s^i) = 0$. Without going into much detail, the strategies Farrell and Maskin [7] use to support WRP payoffs in standard repeated games satisfy conditions *i)* and *ii)* of proposition 5.1. This means that only condition 2 differs from condition *iii)* of proposition 5.1.

Proposition 5.3
If $a \in F^+$ is WRP and condition iii) holds for $R(s^i)$ $(i = 1, 2)$, then $(a_1, f_2(a_1))$, $(f_1(a_2), a_2)$ and (a_1, a_2) are SPE payoffs for sufficiently large $\delta < 1$.

Proof.
The proof that $(a_1, f_2(a_1))$ is an SPE payoff of the bargaining game follows directly from the sufficient and necessary conditions of WRP and applying proposition 5.1. Renumbering the players yields that $(f_1(a_2), a_2)$ is also an SPE payoff.
To sustain (a_1, a_2) as an SPE payoff the strategies in the normal phase have to be adapted such that each player i $(i = 1, 2)$ proposes the contract that yields this player a payoff of R_i^{max} and the other player rejects. As long as players do not reach an agreement (and they never will in this normal phase) the disagreement actions are determined by the sequence of actions $< s(t) >$ with resulting payoff a. If player i unilaterally deviates from the normal phase, then player i's punishment phase is

the same as described in the "one sided offer" strategies. These adapted strategies are also SPE because *i)* the strategies in the punishment phase are SPE and *ii)* each player receives an payoff in the normal phase as if the other player was the player who makes the "acceptable" proposals. As a consequence of proposition 5.1 no player has an incentive to deviate from this adapted normal phase. □

Corollary 5.1 *If $a \in F$ is Pareto efficient and WRP, then a is an SPE payoff for sufficiently large $\delta < 1$.*

Proof. Vector a is Pareto efficient implies that $a = (a_i, f_j(a_i))$. Furthermore, $a \in WRP$ implies that there exists an $s^i \in S$ such that $R_j(s^i) \geq a_j = f_j(a_i)$ and $R_i(s^i) < a_i$. Hence, $R_j(s^i) \geq f_j(a_i) \geq f_j(a_i) + f_j'(a_i)[a_i - R_i(s^i)]$ and a satisfies the condition of lemma 5.1 for $i = 1, 2$. □

Evans and Maskin [7] have proved that generically all games have WRP payoffs that are also Pareto efficient, for sufficiently large $\delta < 1$. Corollary 5.2 states that all WRP payoffs that are also Pareto efficient have to be SPE payoffs of the bargaining model. This implies that for almost all games Γ the limit set of SPE payoffs that can be supported as "one sided offer" strategies in the normal phase has to be large. Therefore, it is generically impossible to obtain a unique SPE outcome. Thus, the model in this paper does not yield the uniqueness result obtained in the alternating offer model (Rubinstein [14]). However, example 6.3 has a unique WRP that is not Pareto efficient and which does not satisfy the conditions of proposition 5.1. As a consequence of corollary 5.1 this means that the set of SPE payoffs constructed in this section can be the empty set. But as said before, this result will only occur in very rare cases and is not general.

6 Examples

The following five examples show the variety of possible results. For the repeated prisoners' dilemma, which satisfies $F^+ = F$ and $\lim_{\delta \to 1} WRP = F$, the full Folk-Theorem (every individually rational payoff can be supported as an SPE) is obtained. The second and third example, a simple Cournot-duopoly and the stone-scissor-paper game (both taken from Farrell and Maskin [7]), which satisfy $F^+ = F$ but not $\lim_{\delta \to 1} WRP = F$, show that the limit sets of propositions 4.1 and 5.1 are strict subsets of F. Example 6.2 shows that proposition 5.1's limit set of SPE payoffs is strictly larger than proposition 4.1's limit set of SPE payoffs. However, example 6.3 shows the opposite result is also possible. Hence, no general result can be derived which states that one of these two limit sets contains the other limit set. Example 6.3 is special because proposition 5.1's limit set is the empty set. As argued in section 5 this result is nongeneric.

For a large class of games Γ it holds that $F^+ \neq F$. As a consequence of proposition 3.2 the limit set of SPE outcomes, as δ goes to 1, is reduced compared to the limit set of SPE payoffs predicted by the Folk-Theorem (Fudenberg and Maskin [9]). The battle-of-the-sexes game illustrates this point. Although the analysis and the examples seem to suggest that non-uniqueness is the general rule, there is a class of games Γ in which the set of SPE payoffs exists of exactly one element. The Pareto efficient frontier for this class of games exists of exactly one element, so that $(R_1^{max}, R_2^*) = (R_1^*, R_2^{max})$ and $F^+ = \{(R_1^{max}, R_2^*)\}$.

Example 6.1 (Prisoners' dilemma)

	L_2	R_2
L_1	(4,4)	(0,5)
R_1	(5,0)	(1,1)

This example is also treated in Sorin [16] and van Damme [4]. Player i's minmax value $v_i = 1$ $(i = 1, 2)$ and the set F^+ coincides with the set F in this example.

Applying proposition 4.2 yields $\hat{s}^e = (L_1, R_2)$ and $g_1(\hat{s}^e) = 1$. The corresponding equilibrium proposals are $\hat{x} = (1, 4\frac{3}{4})$ and $\hat{y}(\hat{s}^e) = (\frac{1}{\delta}, \frac{20\delta - 1}{4\delta})$, which converge to $(1, 4\frac{3}{4})$ as δ goes to 1. Thus, player 1's lowest limit SPE payoff is equal to 1. By symmetry, player 2's lowest SPE payoff is equal to 1. Hence, the limit set of SPE payoffs associated with "immediate agreement" strategies is the whole set F.

For this example it is sufficient to apply proposition 5.3 in order to obtain the whole set of "one sided offer" SPE payoffs. Van Damme [4] has shown that $WRP = F$ for $\delta \geq \frac{1}{4}$. Farrell and Maskin [7] have shown that every $a \in F$ can be sustained as a WRP with the pure strategies $s^1 = (L_1, R_2)$ and $s^2 = (R_1, L_2)$. It can easily be checked that every $a \in F$ and s^i $(i = 1, 2)$ satisfy the condition of lemma 5.1, because

$R_2(s^1) > R_2^{max} \geq f_2(a_1)$ (similarly for s^2). Hence, the limit set of SPE payoffs associated with "one sided offer" strategies is the whole set F.

Farrell and Maskin [7] have argued that this example is a-typical, because the pair of actions $s^1 = (L_1, R_2)$ and $s^2 = (R_1, L_2)$ can sustain every $a \in F$ as an WRP payoff. This example is also a-typical in the context of this paper, because these two pairs of (punishment) actions automatically satisfy the conditions of lemma 5.1 independent of $a \in F$.

Example 6.2 (Cournot-Duopoly)

Consider two firms in a Cournot-duopoly with revenue functions

$$R(s_1, s_2) = \big(s_1(2 - s_1 - s_2), s_2(2 - s_1 - s_2)\big),$$

where $s_i \in [0, 2]$ $(i = 1, 2)$ denotes the quantity produced by player i.

This example is also treated in Farrell and Maskin [7]. Player i's minmax value $v_i = 0$ $(i = 1, 2)$ and the set $F = \{a \in \mathbf{R}^2_+ | a_1 + a_2 \leq 1\}$. Furthermore, player i's best response function $\beta_i^*(s_j) = \frac{1}{2}(2 - s_j)$ $(j = 1, 2$ and $j \neq i)$. Player i's corresponding revenue $R_i(\beta_i^*(s_j)) = \frac{1}{4}(2 - s_j)^2$.

Applying proposition 4.2 (and remark 4.3) yields $\hat{s}^e = (0, \frac{6}{5})$. Thus, $R(\hat{s}^e) = (0, \frac{24}{25})$ and $g_1(\hat{s}^e) = \frac{4}{25}$. The corresponding equilibrium proposals are

$$\hat{x} = \left(\tfrac{1}{5(1+\delta)}, \tfrac{4+5\delta}{5(1+\delta)}\right) \quad \text{and} \quad \hat{y}(\hat{s}^e) = \left(\tfrac{4+\delta^2}{25\delta(1+\delta)}, \tfrac{24\delta^2+25\delta-4}{25(1+\delta)}\right),$$

which converge to $(\frac{1}{10}, \frac{9}{10})$ as δ goes to 1. Therefore, the limit set of SPE payoffs associated with "immediate agreement" strategies is the set $\{a \in F | a_1, a_2 \geq \frac{1}{10}\} \neq F$.

Proposition 5.2 is applied by first deriving the limit set of WRP payoffs and then solving the optimisation problem of proposition 5.2 for the limit set of WRP payoffs. For notational simplicity we only consider strategies that punish player 1. Farrell and Maskin [7] have proved that $a \in WRP$ iff $16a_1 \geq (a_2 + 4a_1)^2$. The interpretation of the vector a (if this vector lies on this curve) is the following. If player 1 deviates from the payoff vector $R(s^1) = (0, a_2)$ (with corresponding actions $s^1 = (0, 1 + \sqrt{1 - a_2})$), then player 1's deviation payoff is exactly a_1 (which is equal to $\frac{1}{4}[2 - (1 + \sqrt{1 - a_2})^2]$).

Consider some vector a on the curve mentioned above, then $R(s^2) = (0, a_2)$ has to satisfy the condition of lemma 5.1, that is $a_2 \geq 1 - 2a_1$, because $f_2(a_1) = 1 - a_1$. Denote a^* as the SPE payoff that yields player 1's infimum SPE payoff, then a^* has to lie on the curve mentioned above and $a_2^* = 1 - 2a_1^*$. Solving yields $a_1^* = \frac{3}{2} - \sqrt{2} < \frac{1}{10}$ (and $s^1 = (0, 2 - 2\sqrt{\frac{3}{2} - \sqrt{2}})$). Hence, the limit set of SPE payoffs associated with "one sided offer" strategies is $\{a \in F | a_1, a_2 > \frac{3}{2} - \sqrt{2}\} \neq F$.

This example shows that there exist SPE payoffs in the bargaining game that are not WRP payoffs in the standard repeated game. For example, every $a \in F$ that satisfies $a_1 > \frac{3}{2} - \sqrt{2}$ and $16a_1 < (a_2 + 4a_1)^2$ (the vector a lies above the curve mentioned above) are SPE payoffs in the bargaining game, but a does not belong to the set of WRP payoffs. On the other hand, this example also shows that not all WRP payoffs in the standard repeated game can be supported as SPE payoffs in the bargaining game. All WRP payoffs $b \in F$ with one coordinate strict smaller than $\frac{3}{2} - \sqrt{2}$, including the Nash equilibrium of Γ, do not belong to the set of SPE payoffs.

However, the most important result is that the limit set of SPE payoffs found by applying proposition 5.2 is strictly larger than the limit set of SPE payoffs found by applying proposition 4.2. This example also belongs to the class of games Bush and Wen [2] consider. This example shows that their claim that the limit set of equilbria is completely characterised by proposition 4.2 is not true.

Example 6.3 (Stone-scissor-paper game)

	L_2	M_2	R_2
L_1	(1,0)	(0,1)	(0,0)
M_1	(0,0)	(1,0)	(0,1)
R_1	(0,1)	(0,0)	(1,0)

This example is also treated in Farrell and Maskin [7]. Player i's minmax value $v_i = \frac{1}{3}$ $(i = 1, 2)$ and the set $F^+ = F = \{a \in \mathbf{R}_+^2 | a_1, a_2 \geq \frac{1}{3}, a_1 + a_2 \leq 1\}$. The unique NE payoff is $(\frac{1}{3}, \frac{1}{3})$ and the corresponding mixed actions are $s^N = ((\frac{1}{3}, \frac{1}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}))$.

Applying proposition 3.1 yields stationary SPE payoffs $x = (\frac{2+\delta}{3(1+\delta)}, \frac{1+2\delta}{3(1+\delta)})$ in all even periods t and $y = (\frac{1+2\delta}{3(1+\delta)}, \frac{2+\delta}{3(1+\delta)})$ in all odd periods t. Both x and y converge to $(\frac{1}{2}, \frac{1}{2})$ as δ goes to 1. Furthermore, $x_1 = y_2 > \frac{1}{2}$.

Applying proposition 4.2 yields mixed actions $\hat{s}^e = ((1,0,0), (\frac{1}{2}, \frac{1}{2}, 0))$, $R(\hat{s}^e) = (\frac{1}{2}, \frac{1}{2})$ and $g_1(\hat{s}^e) = 0$. The corresponding equilibrium proposals are $\hat{x} = (\frac{5}{6(1+\delta)}, \frac{1+6\delta}{6(1+\delta)})$ and $\hat{y}(s^e) = (\frac{2+5\delta-2\delta^2}{6(1+\delta)}, \frac{4+\delta+2\delta^2}{6(1+\delta)})$, which converge to $(\frac{5}{12}, \frac{7}{12})$ as δ goes to 1. Thus, player 1's lowest limit SPE payoff is equal to $\frac{5}{12}$. By symmetry, player 2's lowest limit SPE payoff is equal to $\frac{5}{12}$. Hence, the limit set of SPE payoffs is the set $\{a \in F | a_1, a_2 \geq \frac{5}{12}\} \neq F$.

Farrell and Maskin [7] have shown that the set WRP is a singleton, namely the unique NE payoff $(\frac{1}{3}, \frac{1}{3})$. Applying proposition 5.3 yields that no WRP payoff satisfies the condition of lemma 5.1. Applying proposition 5.2 yields $s^* = \hat{s}^e$. Thus, $R(s^*) = (\frac{1}{2}, \frac{1}{2})$ and $g_1(s^*) = 0$. As a consequence of proposition 5.2 it follows that $a \in F$ is a SPE payoff in "one sided offer" strategies if and only if $a_1 \geq R_1(s^*)+g_1(s^*) = \frac{1}{2}$. This $R(s^*)$ cannot be supported as an SPE payoff using the "one sided offer" strategies of section 5 where player 2 is punished with the stationary SPE, because $g_2(s^*) = \frac{1}{2}$ and $y_2 > \frac{1}{2}$. This implies that player 2 will certainly deviate at the third stage of all even periods.

Renumbering the players and applying proposition 5.2 yields $s^{**} = ((\frac{1}{2}, \frac{1}{2}, 0), (0, 1, 0))$, $R(\hat{s}^{**}) = (\frac{1}{2}, \frac{1}{2})$ and $g_2(\hat{s}^{**}) = 0$. As before, $a \in F$ is an SPE payoff in "one sided offer" strategies if and only if $a_2 \geq R_2(s^{**}) + g_2(s^{**}) = \frac{1}{2}$. Similar reasoning shows that $R(s^{**})$ can not be supported as an SPE payoff, because player 1 will certainly deviate. Hence, the limit set of SPE payoffs for this type of strategy is the empty set. As argued in section 5, this result is not general.

This example and example 6.2 show that it is not possible to prove that one of the two limit sets of SPE payoffs (associated with propositions 4.2 and 5.2) contains the other limit set.

Example 6.4 (Battle of the Sexes)

	L_2	R_2
L_1	(3,1)	(0,0)
R_1	(0,0)	(1,3)

The set F^+ does not coincide with the set F in this example. This game has two pure NE, namely (L_1, L_2) (with payoff (3,1)), (R_1, R_2) (with payoff (1,3)) and one mixed NE $(s_1, s_2) = ((\frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4}))$ (with expected payoff $(\frac{3}{4}, \frac{3}{4})$)

Applying proposition 3.1 yields that $(3, 1)$ and $(1, 3)$ can be sustained as stationary SPE payoffs. Combining these results with proposition 3.2 and the fact that stationary SPE strategies are a special type of "immediate agreement" strategies implies that proposition 4.2 yields that the whole set F^+ is the limit set of SPE payoffs associated with "immediate agreement" strategies.

For the limit set of SPE payoffs found by applying proposition 5.2 the same result holds, because (3,1) and (1,3) are WRP payoffs and these payoffs also satisfy the conditions of proposition 5.3 for every $a \in F^+$.

Note that the mixed actions NE, which is also a WRP payoff, does not satisfy the conditions of proposition 5.3 and can therefore not be sustained as an SPE payoff. Thus, the bargaining game can not only reduce the set of SPE's predicted by the Folk-Theorem but also the set of WRP payoffs.

Example 6.5 (F^+ is singleton)

	L_2	R_2
L_1	(2,2)	(1,0)
R_1	(0,0)	(0,0)

This game is also treated in van Damme [4] and has a unique NE, namely (L_1, L_2), which is Pareto efficient. The Pareto efficient frontier of F is degenerated in the sense that it is the singleton $\{(2,2)\}$. Therefore, $(R_1^{max}, R_2^*) = (R_1^*, R_2^{max}) = (2,2)$ and $F^+ = \{(2,2)\}$. Proposition 3.1 implies that (2,2) can be sustained as a stationary SPE payoff and together with proposition 3.2 this result immediately implies that the limit set of SPE payoffs is the whole set F^+ as δ goes to 1. Note that we only obtain uniqueness in SPE payoffs and not in SPE strategies, because the strategies of proposition 5.1 are also SPE.

7 Related Work

In this section some related work will be discussed. Okada [12] has analysed the same model, except that he assumes that both players use the limiting average criterion to evaluate their stream of payoffs. Okada [12] claims that the Folk-Theorem holds (the set of SPE payoffs is equal to the set F). Unfortunately, this result is not correct, unless $F^+ = F$. If not (as in examples 6.4 and 6.5), then the strictly smaller subset F^+ contains the set of SPE payoffs, because the arguments used in the proof of proposition 3.2 are also valid if the limiting average criterion is used instead of discounting. Secondly, by using the limiting average criterion to evaluate payoffs the bargaining model does not have the *shrinking-cake property.* Without this property even the standard alternating offer model does not have a unique SPE. For the prisoners' dilemma (example 6.1) each Pareto efficient payoff can be obtained as a stationary SPE in the model of Okada [12]. With this multiple of equilibria it is not difficult to construct inefficient SPE payoffs in F.

Bush and Wen [2] analyse a model that is almost identical to the model of this paper. As long as players do not agree, they have to play the static game as in the bargaining model of this paper. The major difference is that in their model players do not bargain over the set F of the static game, but over some fixed surplus $s \in \mathbf{R}_+$. This surplus s satisfies the condition that the line $a_1 + a_2 = s$ lies either strictly above or is tangent to the Pareto frontier of the set F. Bush and Wen [2] give a characterisation result of the set of SPE payoffs and claim that they have derived a lower bound for each player's SPE payoffs for the class of games they consider. Unfortunately, they only consider "immediate agreement" strategies. Example 6.2 (which also satisfies the assumptions Bush and Wen [2] make in case $s = 1$) shows that the set of SPE payoffs that results from "one sided offer" strategies is strictly larger than the set of "immediate agreement" strategies. Hence, the lower bound on the set of SPE's derived by Bush and Wen [2] is only valid for the type of strategies they consider and not for all type of strategies.

The model of Bush and Wen [2] can be seen as a generalisation of the wage bargaining model of Fernandez and Glazier [8] and, Haller and Holden [10] in which one union and one firm bargain over the division of the profit. The static game in the latter model can be viewed as a degenerate static game in which the union has two options ("strike" and "not strike") and the firm has only one action (implement some old wage contract that is exogenously given). The action "not strike" is not only the unique NE of this static game but also the unique WRP equilibrium which happens to be Pareto efficient. In this model the "immediate agreement" strategies yield the largest set of SPE payoffs.

The type of model that is analysed in this paper is also suggested by Cichocki and Stefanski [17] to analyse bargaining between the government and a major national firm, where state variables such as the government deficit and the firm's capital stock are taken into account. Houba and de Zeeuw [11] formalised this idea to a general bargaining model in a dynamic system with a finite number of periods and binding contracts. They also investigate finitely repeated games as a special case and find that for the prisoners' dilemma the bargaining model has a unique SPE in payoffs. Compared with the results for the prisoners' dilemma in this paper (a large set of SPE payoffs) it can be concluded that the set of SPE payoffs is discontinuous with respect to the time horizon. This discontinuity is also known from standard repeated game models.

In the second subsection of section 5 the relation between weakly renegotiation proof equilibria (WRP) for standard repeated games (Farrell and Maskin [7]) and "one sided offer" SPE strategies has been investigated. To focus on WRP seems to be restrictive because several concepts of renegotiation proofness for standard repeated games have been proposed in the literature (Asheim [1], Bernheim and Ray [3], Farrell and Maskin [7], Rubinstein [13]), but actually it is not. The reason is that the concept of WRP is more or less the weakest concept of these concepts. Bernheim and Ray [3] have a concept, called "internally consistency", which is equivalent to the concept of WRP. Furthermore, every strong perfect equilibrium (Rubinstein [14]) is also WRP (but not visa versa). Only the concept introduced by Asheim [1] differs from the concept of WRP.

8 conclusions

In this paper the alternating offer model (Rubinstein [14]) is extended to capture the idea that in many economic situations agents have to interact strategically with each other while negotiating. In this paper this strategic interaction is modelled as a static game in between the bargaining rounds and the subject of the bargaining is an infinite sequence of actions in this static game. The analysis in this paper shows that the strategy space is complex. Therefore, it seems impossible to derive a characterisation result that fully determines the set of equilibrium payoffs (as the common discount factor goes to 1).

In this paper two types of strategies are introduced in order to analyse the model, namely "immediate agreement" strategies (section 4) and "one sided offer" strategies (section 5). For each type of strategies sufficient and necessary conditions are derived under which these strategies are equilibrium strategies. By means of two simple examples (example 6.2 and 6.3) it is shown that it is impossible to derive that one of these two equilibrium sets contains the other set. Therefore, the set of equilibrium payoffs is at least the union of these two (sub)sets of equilibrium payoffs. Although it is possible that some other type of strategy exists that supports equilibrium payoffs that are not contained in this union, we conjecture that this seems unlikely and, therefore, it seems that we do have a characterisation result of the set of equilibrium payoffs.

Whatever the exact set of equilibria is, the previous sections show that the Folk-Theorem (all individually rational payoffs are equilibrium payoffs) may hold, as for example the prisoners' dilemma shows, and may not hold in general, as for example the battle-of-the-sexes game shows. In this last case, introducing bargaining with binding contracts strictly reduces the set of equilibrium payoffs predicted by the Folk-Theorem. Although it has not been shown explicitly, non-uniqueness is the general case for the bargaining game in this paper.

Regarding the two types of strategies introduced, it has been shown that "one sided offer" strategies are closely related to weakly renegotiation proof equilibria (Farrell and Maskin [7]). The basic idea underlying renegotation proofness (Asheim [1], Bernheim and Ray [3], van Damme [4], Farrell and Maskin [7], Rubinstein [13]) is that communication or renegotiations will lead to playing undominated strategies. Although the negotiations over strategies in the repeated game is at the heart of this approach no attempt is made to model this bargaining process. The main characteristic of the bargaining model in this paper is that the bargaining process is explicitly modelled. Although the assumption of binding contracts is not in the spirit of the renegotiation proofness literature, the bargaining model (with binding contracts) shows that inefficient SPE outcomes are possible. This last result seems to contradict the basic assumption underlying renegotiation proof equilibria, namely that renegotiation will lead to playing undominated strategies. This result severely questions the foundations of the renegotiation proofness concept. Further investigations in order to find a complete noncooperative theory of renegotiation proofness in repeated games (in a similar fashion to the Nash program for bargaining theory) is necessary to give a complete answer to this problem. However, one of the problems that arises when the assumption of binding contracts is removed is the problem of "cheap talk" (see Farrell [6]).

Notes

1. To avoid confusion later on we call this game the "static" game instead of the "stage" game, which is sometimes done in the literature on repeated games.

2. The exact rules of the game state that any inefficient contract agreed upon at time t will be replaced at time $t+1$, because it is assumed that there is only one bargaining round per period. However, for the limit SPE payoff, as $\delta \to 1$, of the subgame starting at period t the payoff at time t is irrelevant in the aggregate payoff and can therefore be disregarded.

3. A *stationary* or *history independent* strategy is a function $\sigma_i(h(t,\theta))$ that is constant as long as players have not agreed, which means that for every history $h(t,\theta)$ the stationary strategy prescribes the same action to be undertaken.

4. Given f_2 strictly decreasing this is the smallest subdifferentiable in absolute value.

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