

1990-63

ET

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Serie Research Memoranda

Spatio-Temporal Processes in Dynamic Logit Models

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Research Memorandum 1990-63
November 1990





Abstract

Chaos theory has recently caught the attention in the social sciences for its capability of capturing irregular motions which are endogenously produced in economic and social systems.

However, in many cases major questions are arising concerning the real-world significance of critical parameter values leading to chaos, the testability of the models concerned and the validity of model specifications. This paper focusses the attention on this last issue, in particular on the compatibility between rationality and chaos in a social environment.

Firstly, it will be shown that a population modelled by a dynamic structure of a logit type (i.e., a population maximising a dynamic-stochastic micro utility function in a choice problem) can exhibit chaotic or complex movements in its choices. This finding reinforces previous contributions obtained in the rational expectations literature. However, it also questions the validity of the predictions of dynamic logit models in the presence of chaotic behaviour.

Secondly, the importance of time lag effects - and hence the influence of the past - will be examined by considering a degenerated logit model with delays of two and three generations, thus confirming that lags in population dynamics may lead to erratic behaviour in population movement.

These lessons from higher-order dynamics show that the values of the parameters in critical ranges play a fundamental role in chaotic regimes. Consequently much more emphasis has to be placed upon empirical validation of these models.

1. Introduction

Chaos theory - and in general complexity theory - is one of the scientific areas which have recently received much attention among scientists. The theory of chaos deals with deterministic, non-linear systems which are able to produce dynamic motions of such a nature that they sometimes seem to be completely random. This behaviour is due to their sensitive dependence on initial conditions and critical parameter values. In particular a small change in the initial conditions may act as a catalyst leading rapidly to an enormous change (though all time paths are bounded). Consequently, we observe very different trajectories in the long run, so that precise or plausible predictions are - under certain conditions - almost impossible. Complexity theory includes not only deterministic-chaotic dynamics, but any deterministic dynamics that converges to an attractor set that is not a point (see also Brock, 1988).

The fact that deterministic systems can lead to random behaviour is a fascinating result which has generated a large number of studies and publications, starting from the natural sciences (see for a concise historical review Nijkamp and Reggiani, 1990) and next penetrating the social and economic sciences (see, e.g., the informative surveys by Andersen, 1988; Baumol and Benhabib, 1989; Boldrin, 1988; Brock, 1988; Gleick, 1987; Grandmont, 1986; Kelsey, 1988; Lung, 1988; Prigogine and Stengers, 1984).

On the one hand, chaos theory copes with the important issue of non-linear dynamics together with other new mathematical tools such as bifurcation theory, catastrophe theory, singularity theory, fractal theory, thus emphasizing a distinct mode of behaviour that is qualitatively different from linear modes.

It should once more be emphasized here that the issue of non-linear dynamics of a system is not related to its stochastic properties but exclusively to the way synchronic and diachronic processes are intertwined (see also Lichtenberg and Lieberman, 1983, and Liosatos, 1980). Under certain conditions discontinuities in a system's behaviour may then emerge, which reflects essentially a morphogenesis in the evolution of the system concerned. Morphogenesis may be based on either endogenous forces (e.g., behavioural feedbacks), or exogenous forces (e.g., in case of random shock models or ceilings and floors models) or a combination of both (e.g., regime switching models).

An important characteristic of many dynamic systems is their complex, multidimensional and nested structure. Due to large fluctuations caused by

dissipative structures, such systems become unstable and even exhibit bifurcations (see Turner, 1980). In particular, if various subsystems or modules are intertwined in a non-linear dynamic way (sometimes even with differences in the successive rates of change), unexpected switches in evolutionary patterns may take place (see also Haag and Weidlich, 1983, and Nijkamp, 1990).

On the other hand, chaos theory plays a fundamental role in this context by showing that erratic behaviour can be endogenously produced in many deterministic systems (i.e., when stochastic shocks are eliminated). Significant advances in the application of chaos theory can be found in economics and geography. Important theoretical developments in the contributions developed in economics can amongst others be found in the following specific fields:

- cobweb models (Chiarella, 1988)
- growth and business cycle theory (Balducci et al., 1984; Benhabib and Day, 1982; Boldrin, 1988; Day, 1982; Funke, 1987; Guckenheimer et al. 1977; Grandmont, 1985, 1986; Stutzer, 1980)
- long waves analysis (Nijkamp, 1987a; Rasmussen et al. 1985; Sterman, 1985)
- R&D analysis (Baumol and Wolff, 1983; Nijkamp and Poot, 1989)
- consumer behaviour (Benhabib and Day, 1981)
- duopoly theory (Rand, 1978; Dana and Montrucchio, 1986)
- economic competition (Deneckere and Pelikan, 1986; Ricci, 1985)
- international trade (Lorenz, 1987)

To the same extent interesting applications of chaos theory related to geography and regional science have recently emerged (see also Nijkamp and Reggiani, 1990). Examples are:

- regional industrial evolution (White, 1985)
- urban macro dynamics (Dendrinos, 1984)
- spatial employment growth (Dendrinos, 1986)
- relative population dynamics (Dendrinos and Sonis, 1987)
- spatial competition and innovation diffusion (Sonis, 1986, 1988)
- migration systems (Reiner et al. 1986; Mosekilde et al., 1988)
- urban evolution (Nijkamp and Reggiani, 1990)

From all these studies various important issues are emerging. On the one hand, according to Sterman (1988) << the significance of the results hinges in large measure on whether the chaotic regimes lie in the realistic

region of parameter space or whether they are mathematical curiosities never observed in a real system >> (p. 148). On the other hand, there are major questions concerning the validity of model specifications (i.e., are model specifications compatible with plausible economic hypotheses) and of testability of model results (i.e., are model results - qualitatively or quantitatively - justifiable from possible non linear patterns in the underlying data set).

The lack of empirical contents (owing to unavailability of data and measurement errors in data) often forces researchers to solve the latter issue by means of simulation experiments which study the heuristics with which people manage a complex dynamic environment (see, e.g., Sterman, 1988 with reference to a stock management problem). In these simulations the parameters of the proposed heuristic are econometrically estimated.

It should be noted that the validity of model specifications is often correlated to the decision rules postulated in some models of human systems: in particular we recall here the interesting debate on the compatibility between rationality and chaos which has produced as an interesting result that chaotic solutions can occur in the presence of impatient economic agents (see Boldrin and Montrucchio, 1986) or incomplete, imperfect competitive markets (Boldrin and Woodford, 1988). As a consequence it results that << chaos has an interesting implication for the rational expectations literature. If the economy happens to be in a chaotic regime, then, even if economic agents know perfectly how the economy functions, they are unable to predict its behaviour, except probabilistically >> (Pohiola, 1981, p. 37).

This paper focusses the attention on the latter issue by extending the dilemma of rationality/chaos to any population with rational intertemporal choices. In particular it will be shown that a dynamic-macro-behaviour, derived from the maximization of a stochastic micro utility function as formalized in logit models, can exhibit chaotic or complex movements. Since dynamic logit models describe the choice probabilities among discrete alternatives the fields of application can be manifold, such as migration analysis, transportation analysis, residential choice analysis, industrial location analysis (see, e.g., Fischer et al., 1990, Hauer et al., 1990 and Nijkamp, 1987b). Since complex behaviour seems to emerge for particular values of the marginal utility function, the present paper also aims to treat the interesting issue of the relationships between (random) utility theory and chaos theory and consequently the predictive power of discrete choice models (derived from the above mentioned random utility theory).

Furthermore, it will also be shown in this paper how the introduction of time lags in a dynamic logit model may exhibit a more complex behaviour under specific conditions of the utility function.

It should be noticed that in the sequel we assume some elementary knowledge on the mathematics of chaos and non-linearity, so that extensive mathematical definitions can be dropped. For further details we recommend some standard references, such as Collet and Eckmann (1980), Devaney (1986), Guckenheimer and Holmes (1983), Iooss (1979) and Poston and Stewart (1986).

2. Chaos in Models of Population Movement of a Logit Type

It will be shown in this section that a discrete choice model of a logit type can exhibit chaotic behaviour - under some conditions of the utility function, as it can be transformed to a generalized logistic growth model with predator-prey properties of competing species.

We will consider the following dynamic logit model:

$$P_{j,t} = \exp(u_{j,t}) / \sum_{\ell} \exp(u_{\ell,t}) \quad (2.1)$$

in which $P_{j,t}$ represents the share of the population choosing a given distinct alternative j ($j=1, \dots, \ell, \dots, J$) at time t and $u_{j,t}$ the utility of choosing j at time t .

Model (2.1) is a time-dependent version of a static logit model (see Heckman, 1981). It is noteworthy that it can also emerge as a solution of an optimal control spatial interaction model, whose objective function is a cumulative entropy function (see Nijkamp and Reggiani, 1988a; 1988b).

Let us first consider the rate of change of $P_{j,t}$ with respect to time (i.e., $dP_{j,t}/dt$):

$$\frac{dP_{j,t}}{dt} = \dot{P}_{j,t} = \frac{d}{dt} [\exp(u_{j,t}) / \sum_{\ell} \exp(u_{\ell,t})] \quad (2.2)$$

Expression (2.2) leads after some simple computational exercises to:

$$\dot{P}_{j,t} = \dot{u}_{j,t} P_{j,t} (1 - P_{j,t}) - P_{j,t} \sum_{\ell \neq j} \dot{u}_{\ell} P_{\ell,t} \quad (2.3)$$

where $\dot{u}_{j,t} = du_{j,t}/dt$ represents the time rate of change of $u_{j,t}$. The utility function may assume various forms, and of course it is interesting to explore in more detail possible trajectories of $\dot{u}_{j,t}$. The second term of (2.3) represents interaction effects which may affect clearly the dynamic trajectory of expression (2.3). The first term in (2.3) contains an

expression representing the well-known continuous time logistic equation depicting logistic growth of the population P_j based on the Verhulst equation. In particular system (2.3) represents a prey-predator system (belonging to the general family of Lotka-Volterra equations¹) with limited prey (P_j); P_ℓ can be interpreted as a predator whose influence will be the reduction of population P_j through the parameter u_ℓ . Now we can approximate equation (2.3) in discrete time by considering a unit time period as follows:

$$P_{j,t+1} - P_{j,t} \approx (u_{j,t+1} - u_{j,t}) P_{j,t} (1 - P_{j,t}) - P_{j,t} \sum_{\ell \neq j} (u_{\ell,t+1} - u_{\ell,t}) P_{\ell,t} \quad (2.4)$$

Note that for the system (2.4) to be consistent ($0 \leq P_j \leq 1$) one must have:

$$|P_{j,t+1} - P_{j,t}| \leq 1 \quad (2.5)$$

or:
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$$|(u_{j,t+1} - u_{j,t}) P_{j,t} (1 - P_{j,t}) - P_{j,t} \sum_{\ell \neq j} (u_{\ell,t+1} - u_{\ell,t}) P_{\ell,t}| \leq 1 \quad (2.6)$$

or:

$$|\alpha_j P_{j,t} (1 - P_{j,t}) - P_{j,t} \sum_{\ell \neq j} \alpha_\ell P_{\ell,t}| \leq 1 \quad (2.7)$$

Now, from (2.7) at the limits, i.e., if (2.5) can hold as equalities, one must have:

$$\alpha_j P_{j,t} (1 - P_{j,t}) = 1 + P_{j,t} \sum_{\ell \neq j} \alpha_\ell P_{\ell,t} \quad (2.8.1)$$

and:

$$\alpha_j P_{j,t} (1 - P_{j,t}) = -1 + P_{j,t} \sum_{\ell \neq j} \alpha_\ell P_{\ell,t} \quad (2.8.2)$$

In other words, from (2.8.1):

¹ The relevance of ecological models of a Lotka-Volterra type in spatial analysis has in recent years often been shown (see among others, Dendrinos and Mullally, 1985 and Sonis, 1983).

In other words, from (2.8.1):

$$\alpha_j^* P_{j,t} = 1 + P_{j,t} \sum_m \alpha_m^* P_{m,t} ; m = 1, 1, \dots, J \quad (2.9.1)$$

for any set of: $0 < P_{j,t} < 1 ; \sum_j P_{j,t} = 1$

and from (2.8.2):

$$\alpha_j^{**} P_{j,t} = -1 + P_{j,t} \sum_m \alpha_m^{**} P_{m,t} \quad (2.9.2)$$

for any set of: $0 < P_{j,t} < 1 ; \sum_j P_{j,t} = 1$

Thus, there are limits $(\alpha_j^*, \alpha_j^{**})$ (i.e., floors and ceilings) what the α_j 's must obey, for the system to be consistent in the case of $|P_{j,t+1} - P_{j,t}| = 1$. In turn, the above implies that in this particular case there may not be for all j 's such real thresholds; for any arbitrary (but consistent) set of P_j 's (that is, being positive and their sum equal to one) the systems in (2.9.1) and (2.9.2) may not have solutions (i.e., there may not exist real values for the α_j 's which can produce consistent P_j 's).

It is now interesting to compare the first part of (2.4) with the standard discrete logistic growth model for a biological population X_t ($X_t < 1$):

$$X_{t+1} = N X_t (1 - X_t) \quad (2.10)$$

where N is a parameter reflecting the growth rate ($0 < N < 4$).

By deleting for the time being the second (interaction) term of (2.4), and by assuming a constant utility increase (i.e., $u_{j,t+1} - u_{j,t} = \alpha_j$), we find a degenerated case which is equivalent to (2.10), when $\alpha_j = N - 1$. Then we can rewrite (2.4) as follows:

$$P_{j,t+1} = N P_{j,t} \left(1 - \frac{N-1}{N} P_{j,t}\right) \quad (2.11)$$

It is evident that if we make the transformation:

$$X_{j,t} = P_{j,t}^{(N-1)/N} \quad (2.12)$$

equation (2.11) can be written in the canonical form (2.10) (see also Wilson, 1981). Model (2.10) has been thoroughly investigated by May (1976).

This author showed that if:

$$1 < N < 3 \quad (2.13)$$

the fixed point $(1 - \frac{1}{N}, 1 - \frac{1}{N})$ is an attractor and the system will move towards this stable point². For $N=3$ the system bifurcates and gives a cycle of period 2. Next, for the range of values:

$$3 < N < 4 \quad (2.14)$$

successive bifurcations give rise to a cascade of period doublings in the range $3 < N < 3.8...$ In particular for the value $N = 3.8...$ a fixed point of period three arises giving rise to a chaotic regime, according to the famous statement of Li and Yorke (1975) "Period three implies chaos". Further details can be found in May (1976), Collet and Eckmann (1980) and others.

It is thus easily seen that the degenerated dynamic (discrete) logit model as specified in (2.11) belongs to the family of May models.

Therefore, if we map equation (2.11) in the plane (P_j, N) , we get exactly the same type of bifurcations as in the May model leading to a fixed period of period three (and hence to a chaotic behaviour), the only difference being the upper limit greater than 1 (see Fig. 1).

² For the definition of a fixed point, an attractor and other terms related to non-linear analysis, see also Nijkamp and Reggiani (1990).

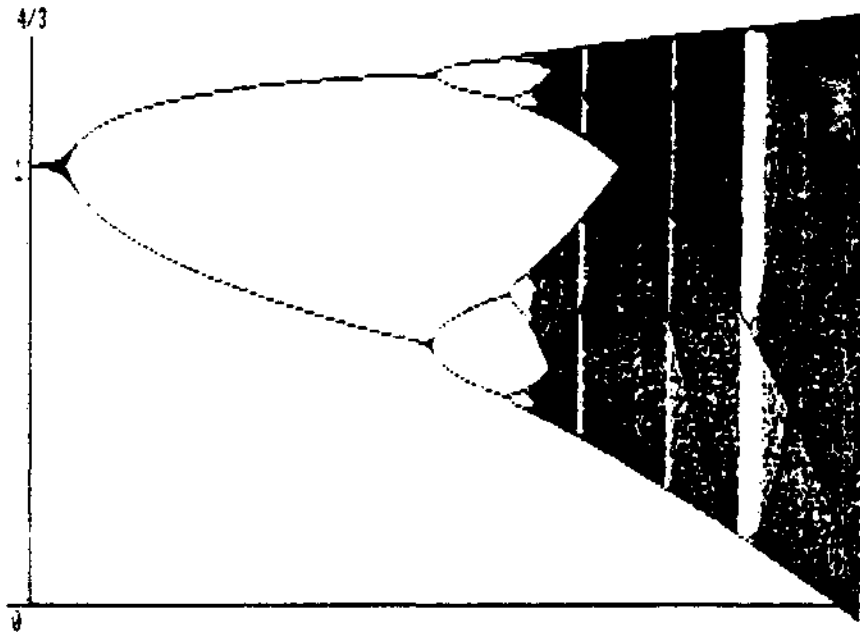


Fig. 1 Bifurcation Diagrams of a Degenerated Logit Map
($2.95 < N < 4$). Y-axis = P_j ; X-axis = N

However, this diagram shows that for $N \geq 3$ the system bifurcates and gives rise to unfeasible values of P_j (in particular $P_j > 1$). In this case one would have to switch to values of $P_j < 1$ (for instance, by including a quadratic penalty function when P_j approaches the upper limit 1 or the lower limit 0), thus causing sudden jumps in the system's trajectory.

The straightforward conclusion is that when $3 < N < 4$, the discrete degenerated logit map (2.11) enters an unstable regime; in other words when the growth of utility with respect to time (related to the first term of the dynamic logit model (2.4)) is less than ≈ 2 , we have stable or periodic solutions in choosing alternative j ; when $2 < \alpha_j < 3$, chaotic behaviour emerges, characterized by unpredictable movements which are hardly foreseeable at the outset. Clearly, if we would use the complete model specified in (2.4), specific alternative trajectories may emerge, depending on the evolution of the competing choice probabilities in this interactive system. In particular, it is plausible that chaotic movements arise, since it has been shown that a discrete system of the Lotka-Volterra type leads to a complex behaviour with strange attractors (see Peitgen and Richter, 1986). The evaluation of the effect of the interaction term in the formulation (2.4)

has to be carried out by means of simulation experiments.

In particular, if we consider the case of three differential equations for system (2.4) (i.e., a dynamic choice problem among three alternatives), it is easy to see that again chaotic movements emerge for high values of the marginal utility functions α_j . Some results of simulation experiments will now briefly be discussed. In particular we will consider the dynamic logit model (2.4) both in relative terms (i.e. by inserting the additivity condition $\sum_{\ell} P_{\ell} = 1$) and in absolute terms (in other words, by examining the equivalent spatial interaction model).

2.1 Temporal processes in logit models

We will present here some results showing the emergence of stability/instability in the dynamic logit models (2.4) (in relative terms) with respect to particular values of the parameter α_j and of the initial conditions.

a) Stable Behaviour

In this simulation we have considered the following parameter values:

$$\alpha_1 = 1 \quad ; \quad \alpha_2 = 0.9 \quad ; \quad \alpha_3 = 1.1$$

with the following initial conditions:

$$P_1 = P_2 = P_3 = 0.33$$

Figure 2 shows a clear stable pattern in the long run.

It is interesting to note the emergence of stable behaviour even for increasing values of α_j . If we consider, for example, the following values:

$$\alpha_1 = 3.0 \quad ; \quad \alpha_2 = 2.85 \quad ; \quad \alpha_3 = 2.88$$

with the same initial conditions:

$$P_1 = P_2 = P_3 = 0.333$$

we still obtain stable trajectories in the long run (see Fig. 3).

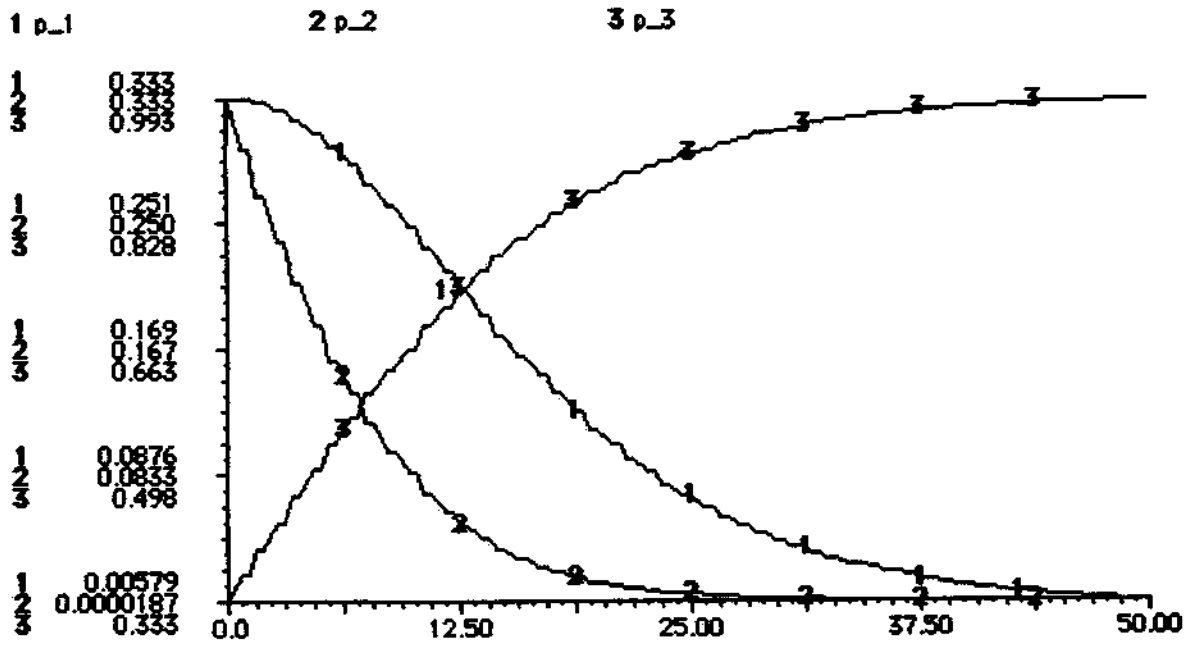


Fig. 2 Stable Behaviour for a Dynamic Logit Model.

Y-axis = P_1, P_2, P_3 ; X-axis = time.

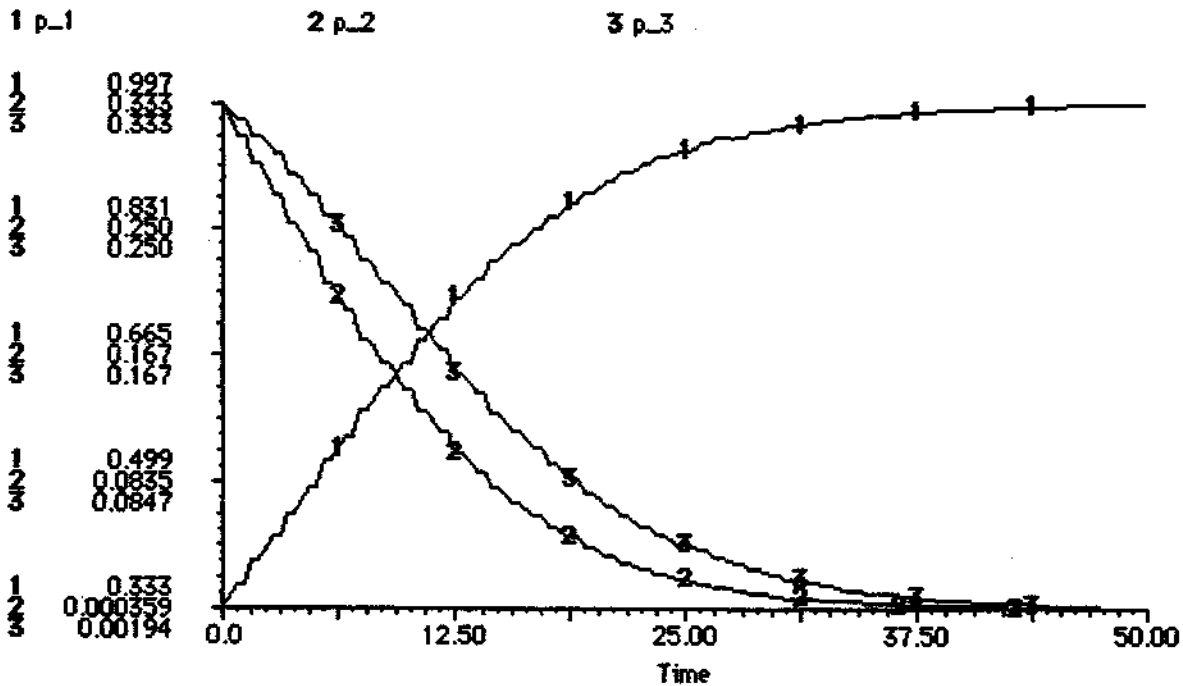


Fig. 3 Stable Behaviour for Increasing Values of the Parameters. Y-axis = P_1, P_2, P_3 ; X-axis = time.

b) Oscillating Behaviour

For this simulation we have assumed higher parameter values, as follows:

$$\alpha_1 = 4.0 \quad ; \quad \alpha_2 = 3.5 \quad ; \quad \alpha_3 = 6$$

with the usual initial conditions:

$$P_1, P_2, P_3 = 0.333$$

Fig. 4 shows a clear oscillating pattern for the whole time period considered.

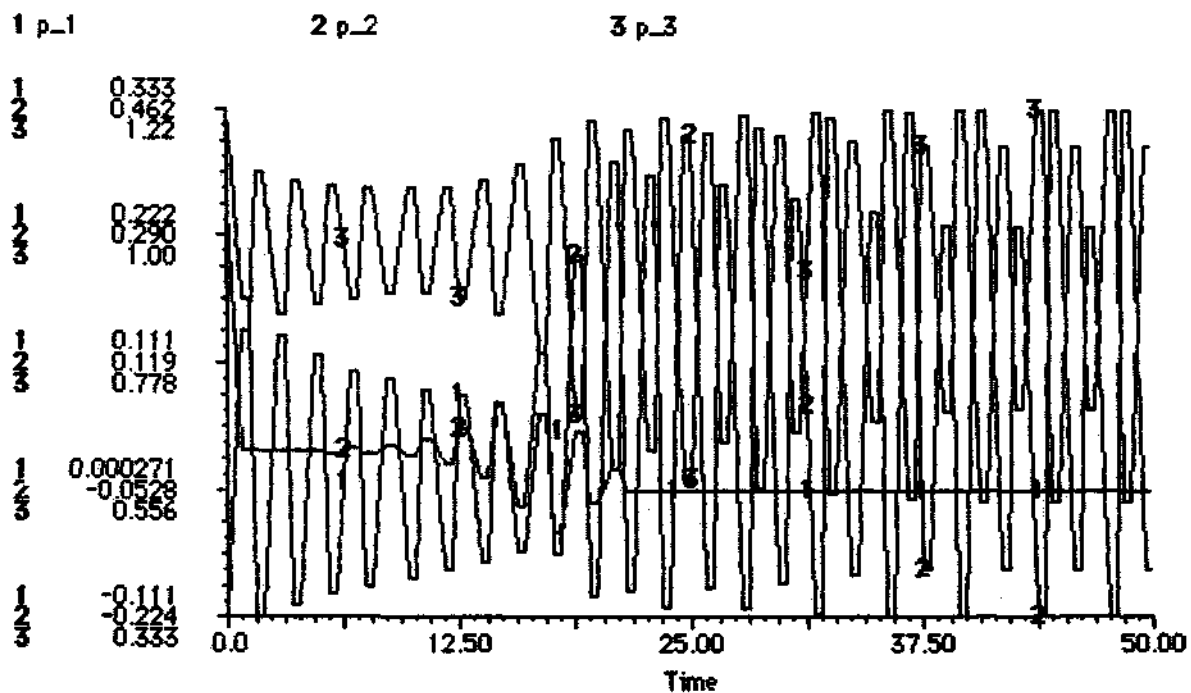


Fig. 4 Oscillating Behaviour for a Dynamic Logit Model.

Y-axis = P_1, P_2, P_3 ; X-axis = time.

We can also observe the sensitivity of model (2.4) with respect to the initial conditions by varying, in this last example, the initial values of the variables. If we assume, for example, $P_1 = 0.5$; $P_2 = 0.2$; $P_3 = 0.3$, we again obtain an oscillating pattern, but of different character (see Fig. 5).

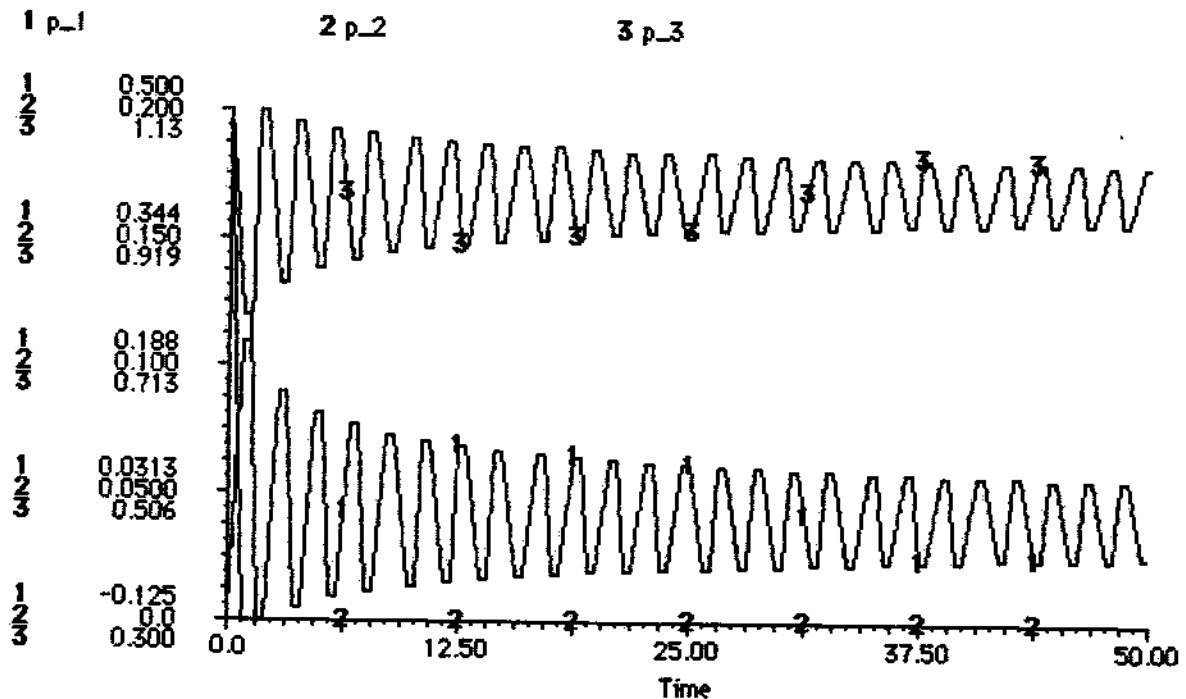


Fig. 5 Oscillating Behaviour in a Dynamic Logit Model
by Varying the Initial Behaviours.
Y-axis = P_1 , P_2 , P_3 ; X-axis = time.

From the previous examples it is easy to observe the emergence of unstable oscillating movements (implying negative values or values greater than 1) by increasing the value of the parameter α_j . Note that we could avoid these unfeasible values by using an exponential logistic function, as specified by May (1974), and by Wilson (1981), in order to avoid unfeasible values of P_j .

It is interesting now to investigate the evolution of the spatial interaction model (2.4) (i. e., by considering the absolute value of the population P_j).

This will be the subject of the next sub-section. In particular we will show - for the equivalent spatial interaction model (2.4) - the emergence of chaotic patterns, even for relatively low values of the parameter α_j .

2.2 Temporal processes in spatial interaction models

In this section some simulation experiments related to the formulation (2.4) in absolute terms will be considered (in other words, the equivalent singly-constrained spatial interaction model).

a) Stable Behaviour

In order to compare the dynamic trajectories of the model (2.4), in relative and absolute terms, in this simulation we have assumed the same parameter values as considered in the first example in 2.1 a), i.e.:

$$\alpha_1 = 1 \quad ; \quad \alpha_2 = 0.9 \quad ; \quad \alpha_3 = 1.1$$

with the following initial conditions:

$$P_1 = P_2 = P_3 = 0.33$$

Figure 6 shows clearly a pattern of stable behaviour in the long run similar to the one illustrated in Fig. 2.

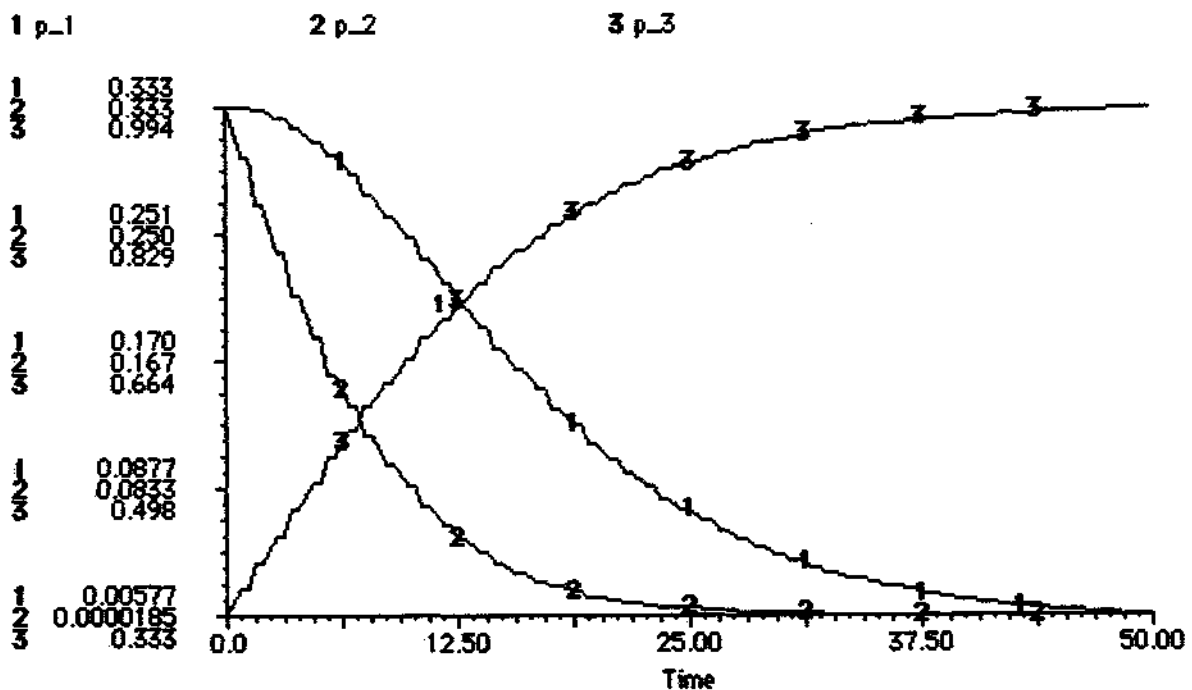


Fig. 6 Stable Behaviour for a Dynamic Spatial Interaction Model.

Y-axis = P_1, P_2, P_3 ; X-axis = time.

b) Chaotic Behaviour

For this simulation we have assumed the parameter values related to the second example in 2.1 a), i.e.:

$$\alpha_1 = 3.0 \quad ; \quad \alpha_2 = 2.85 \quad ; \quad \alpha_3 = 2.88$$

with the same initial conditions:

$$P_1 = P_2 = P_3 = 0.33$$

It is clear from Figure 7 that oscillating trajectories occur for the three populations of the spatial interaction model (2.4), while in the parallel case of a logit model we get stable behaviour (see Fig. 3). It is also interesting to notice the importance of the parameter values for α_j^* , since they seem to govern the amplitude of the trajectories. For example,

if we consider the case of equal marginal utility functions (e.g., $\alpha_1 = \alpha_2 = \alpha_3 = 2.98$), with the same initial conditions, we get the same type of chaotic trajectory for all three populations (see Figure 8).

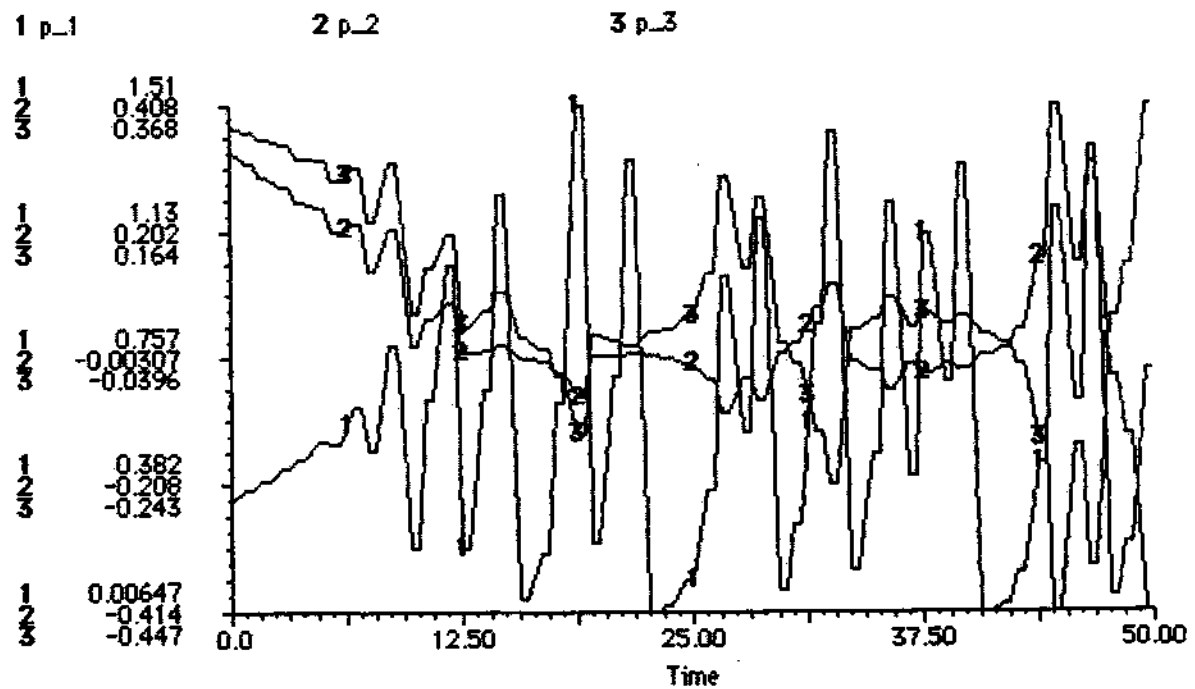


Fig. 7 Chaotic Behaviour for a Dynamic Spatial Interaction Model with Different Utility Increases

Y-axis = P_1, P_2, P_3 ; X-axis = time

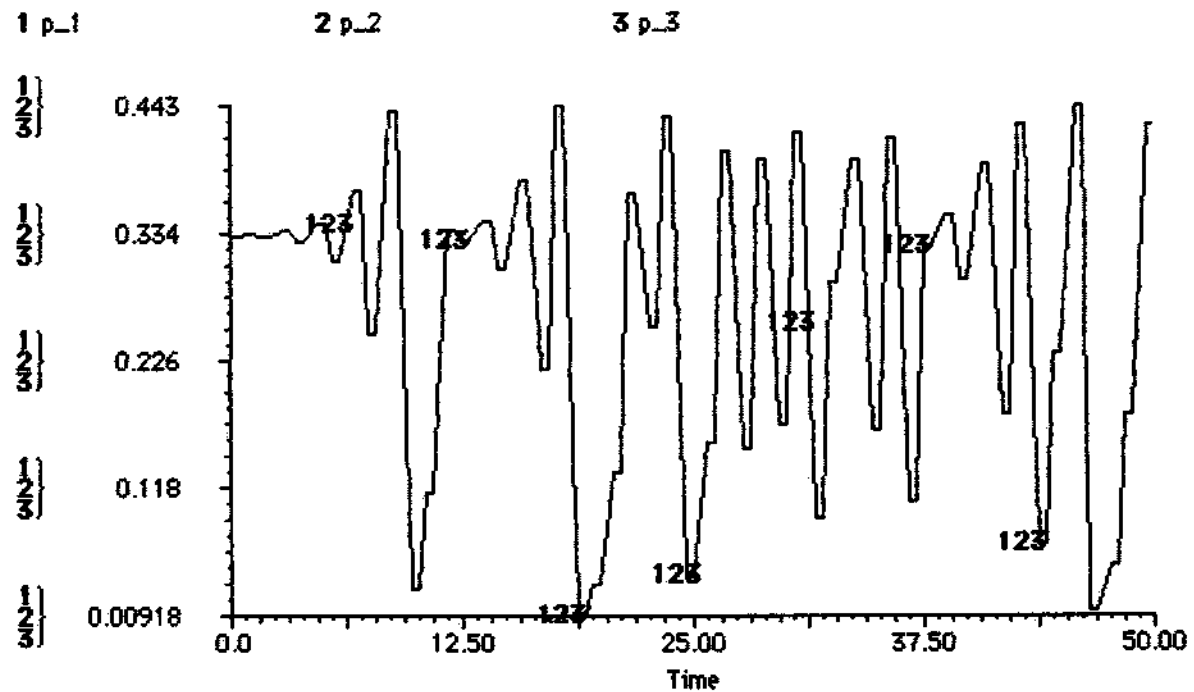


Fig. 8 Chaotic Behaviour for a Dynamic Spatial Interaction Model
with Equal Utility Increases
Y-axis = P_1 , P_2 , P_3 ; X-axis = time

It is also interesting to evaluate the impact of the initial conditions on these chaotic patterns. By assuming, for example, the following initial conditions:

$$P_1 = 0.5 \quad ; \quad P_2 = 0.3 \quad ; \quad P_3 = 0.2$$

with the parameter values previously considered:

$$\alpha_1 = 3.0 \quad ; \quad \alpha_2 = 2.85 \quad ; \quad \alpha_3 = 2.88$$

we obtain an intermittency pattern (see Fig. 9). In other words a chaotic behaviour arises in the long run after a period which seems to tend to a stable behaviour.

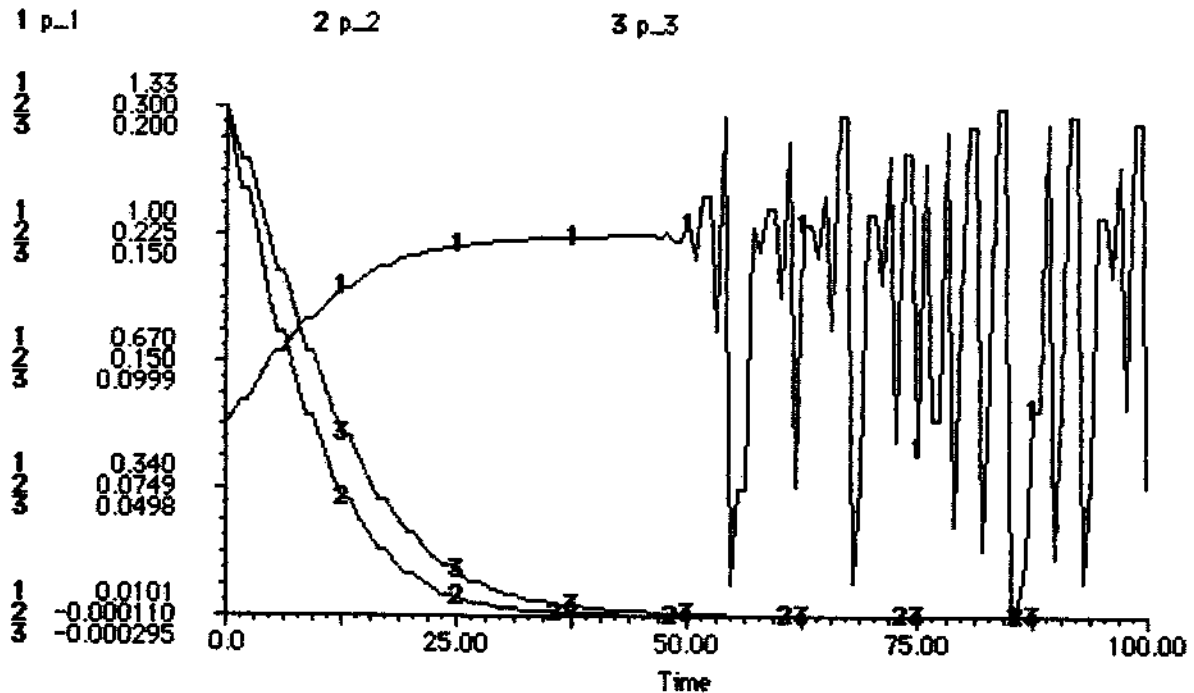


Fig. 9 Intermittency Pattern in a Spatial Interaction Model.

Y-axis = P_1 , P_2 , P_3 ; X-axis = time

Thus it may be concluded that chaotic behaviour seems to emerge in a dynamic logit model, both in degenerated and non-degenerated cases of interaction effects. In the latter case the dynamics of a logit model can be interpreted in the light of a prey-predator system in which the growth parameters (i.e., the marginal utility functions) play a fundamental role for the emergence of chaos. In this context the importance of the speed of change of these parameters has to be given due attention, since some critical parameter values can lead to a chaotic movement with unpredictable values for the successive populations. It should also be noted that this result is in agreement with Dendrinos and Sonis' (1989) analysis, related to the chaotic properties of the universal map of socio-spatial dynamics, since a logit formulation is a particular case of this universal map. Thus our results also show the close connections with recent advances in the area of socio-spatial dynamics.

3. Chaotic Evolution in Population Models of a Delayed Logit Type

In this section the dynamic behaviour of a delayed logit model will be examined. In other words, we will model the growth of the population share P_j (choosing option j) whose ability to grow in any given time span is governed by the population in the previous time span. First we assume for the time being again that the interaction term of (2.4) is deleted and that utility has a constant increase (i.e., $\alpha_j = N-1$). Then we transform equation (2.6) in the following way:

$$P_{j,t+1} = N P_{j,t} \left(1 - \frac{N-1}{N} P_{j,t-s}\right) \quad (3.1)$$

Clearly, in population biology terms, the new family of degenerated discrete logit models (3.1) contains a non-linear term regulating the population size with a time delay of s generations.

These models are indeed most interesting if they are compared with those emerging from the standard delayed logistic equation for a biological population P (see Maynard Smith, 1968):

$$P_{t+1} = N P_t (1 - P_{t-s}) \quad (3.2)$$

where P_t represents the population density in the t^{th} generation and N is a parameter reflecting the growth rate. Let us now first consider the usual case where $s=1$, i.e.,

$$P_{t+1} = N P_t (1 - P_{t-1}) \quad (3.3)$$

which also includes the following degenerated logit model:

$$P_{j,t+1} = N P_{j,t} \left(1 - \frac{N-1}{N} P_{j,t-1}\right) \quad (3.4)$$

Model (3.3) has been investigated by several authors (see for example Aronson et al., 1982; Lauwerier, 1986; Pounder and Rogers, 1980).

In particular it has been shown that the fixed point $\left(\frac{N-1}{N}, \frac{N-1}{N}\right)$ related to (3.3) is stable for $1 < N \leq 2$. As N passes through the value 2, this fixed point loses stability via a Hopf bifurcation, giving rise to a chaotic regime. It is easily seen that the same type of stability occurs in

model (3.4), the only difference being the value of the non-trivial fixed point. Consequently the parameter values showing a chaotic attractor are $N > 2$ for both the degenerated delayed logit model (3.4) and the delayed logistic model (3.3). It is also interesting to note that equation (3.3) spawns for $N = 2.27$ a point of homoclinic tangency (see Pounder and Rogers, 1980), leaving a strange attractor whose shape is quite similar to the pictures obtained by Hénon (1976) (See Fig. 10). Consequently, we can infer the conclusion that a degenerated delayed logit model of type (3.4) can display a fractal character.

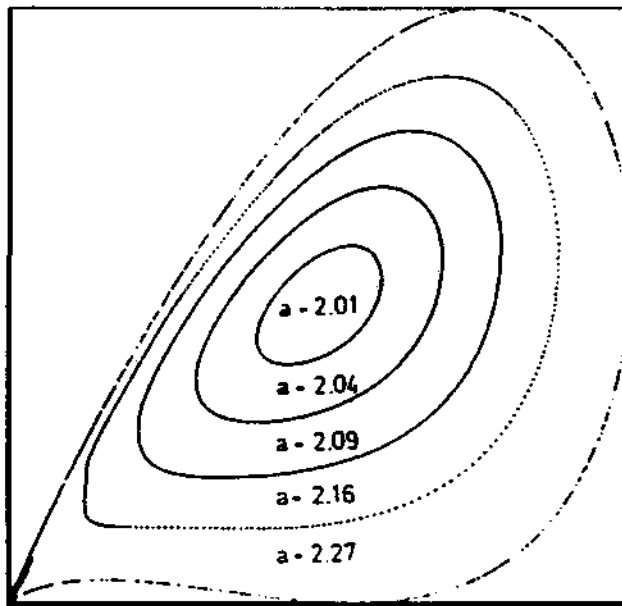


Fig. 10 The Strange Attractor for the Degenerated Delayed Logistic Map (3.3)

Source: Aronson et al., (1982, p. 309)

Y-axis = P_{t+1} ; X-axis = P_t

If we observe now the bifurcation diagrams (see Fig. 11) emerging from model (3.4), we may conclude that also in the degenerated delayed logit model (3.4) a chaotic regime appears for $2 < N < 2.27$.

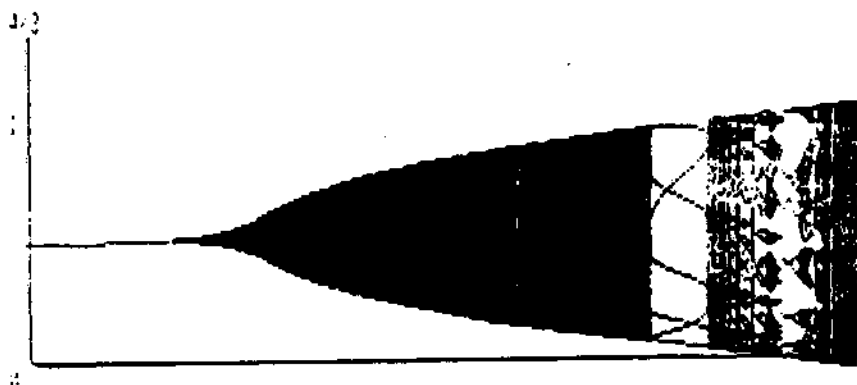


Fig. 11 Bifurcation Diagrams for a Degenerated Delayed Logit Map of Type (3.4)

(1.9 < N < 2.27) Y-axis = $P_{j,t+1}$; X-axis = N

It should also be noted that - in our specific case of a degenerated delayed logit model of type (3.4) - the chaotic regime means again that values of P_j greater than 1 lead apparently to unfeasible system's behaviour (see also Section 2).

In conclusion, it can be easily seen that for the parameter values $0 < N < 1$, system (3.4) embodies stability in its fixed point (0,0), while for $1 \leq N < 2$, system (3.4) represents stability in its fixed point (1,1). Consequently, in a degenerated delayed logit model more restrictive conditions for the parameter N emerge (in comparison to the non-delayed logit model (2.6)) in order to have a stable behaviour.

It is interesting to note that this concluding remark is even more valid in the presence of longer time delays. Let us consider, for example, the delayed logit model (3.1) with $s=2$, i.e., with a delay of one more generation:

$$P_{m,t+1} = N P_{j,t} \left(1 - \frac{N-1}{N} P_{j,t-2} \right) \quad (3.5)$$

Model (3.5) produces bifurcations diagrams (see Figure 12) that are more complex than the ones illustrated in Figure 11, which is related to model (3.4) displaying a shorter delay.

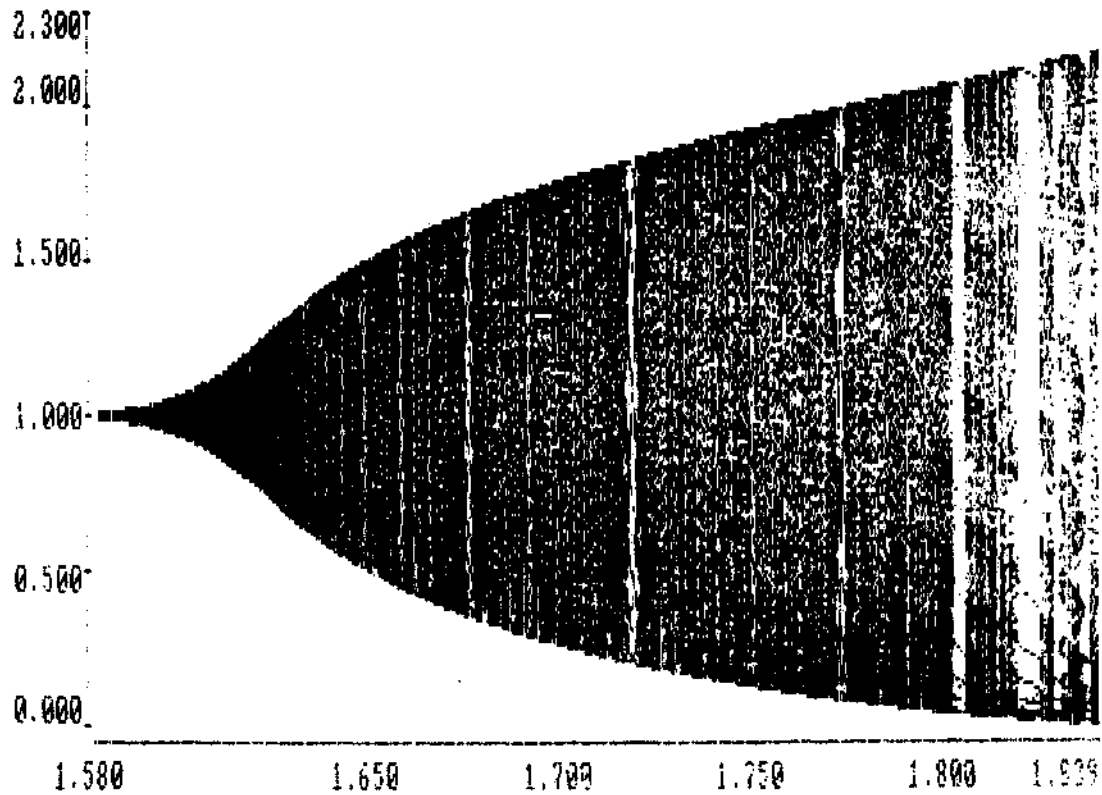


Fig. 12 Bifurcation Diagrams for a Degenerated Delayed Logit Map of Type (3.5)

($1.6 < N < 1.84$) Y-axis = $P_{j,t+1}$; X-axis = N

From Figure 12 we can also observe that the values of N displaying complex behaviour are decreasing. For the sake of simplicity we can summarize the range of N leading to stability for degenerated logit models considered here, as follows:

Logit models	Stability or periodic behaviour	Complex behaviour with bifurcations
one time delay	$0 < N < 3$	$\sqsubset 3 < N < 4$
two time delays	$0 < N < 2$	$\sqsubset 2 < N < 2.27$
three time delays	$0 < N < 1.6$	$\sqsubset 1.6 < N < 1.84$

Table 1. Ranges of the Parameter N Leading to Stability for the Degenerated Logit Models

From Table 1 we can infer the conclusion that the influence of the past leads to less opportunities of getting stable behaviour. Furthermore, by

increasing the delay effects, the emerging unstable behaviour appears to be still more complex. In fact if we 'blow up' the last part of the diagram illustrated in Fig. 12, we will even observe a big window of period ≤ 12 (while in Fig. 11 we would observe a big window of period ≤ 7), followed by a sequence of bifurcations likely leading again to a strange attractor around the value of $N=1.839$ (see Fig. 13). However in this case the strange attractor will be three-dimensional; we may plausibly assume that its projection in a bi-dimensional plane belongs to Hénon's family.

The result is in agreement with the analysis carried by Maynard Smith (1974) who has underlined the presence of oscillating behaviour in populations with time lags of type (3.2) for values of s much greater than $1/N$.

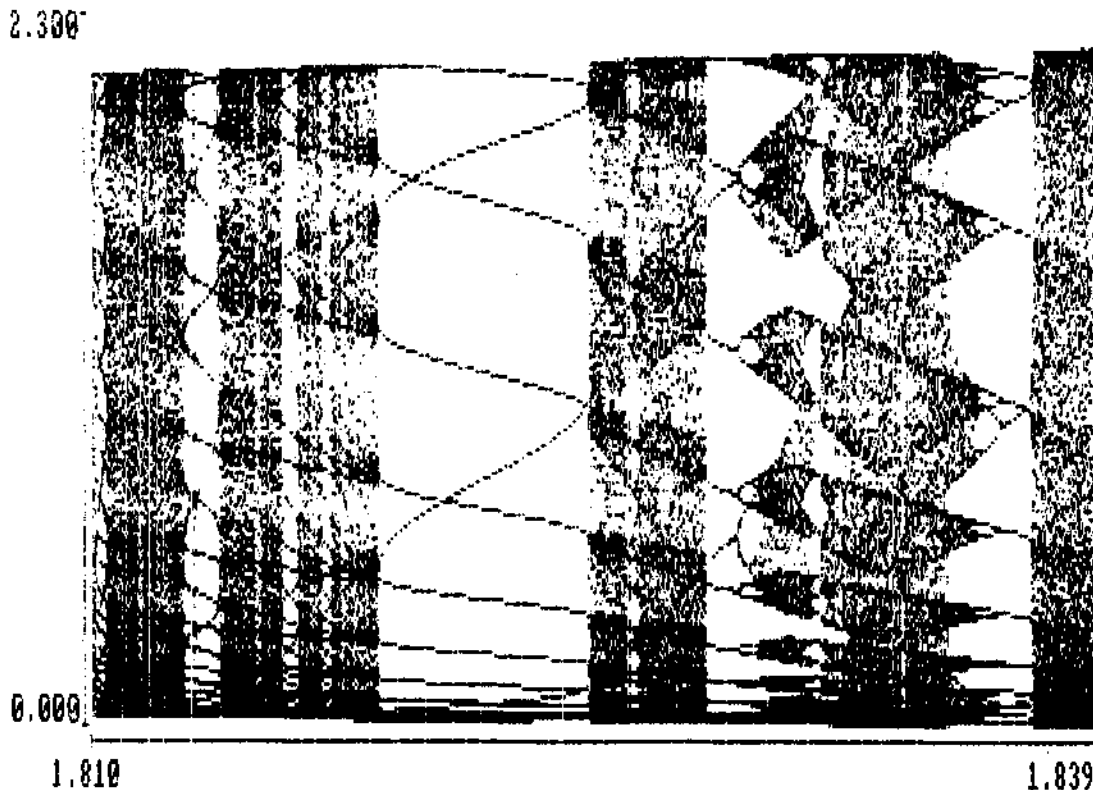


Fig. 13 A Blow-up of Fig. 12 ($1.810 < N < 1.839$)

Y-axis = $P_{j,t+1}$; X-axis = N

In conclusion, in our particular case of a model of population movements driven by the choice for a particular spatial alternative, we may conclude that when the influence of the past is increasing, the actors produce an unstable oscillating behaviour, certainly for high values of their marginal utility function $\alpha_j = N-1$.

Once again (see also Nijkamp and Reggiani, 1989) the introduction of

time lags in dynamic equations of population movements shows a richer spectrum of complex behaviour. Parallel to the previous section, we can also conclude that the effect of interaction terms in a general delayed logit model is a higher de-stabilization of the system concerned (in comparison to the non-delayed logit system), at least for high values of the utility increase α_j .

4. Conclusion

Our analysis has shown the possibility of chaotic and complex behaviour for both a dynamic logit and a delayed logit model. This analysis deals with the issue of the relationships between rationality and chaos. In particular since logit models are derived from the maximization of a stochastic utility function (in which the measurement errors, the imperfect information flows, etc. are included), this result is compatible with previous arguments in economic theory which underline the emergence of chaos in the presence of impatience and imperfect knowledge in the decision-makers. In this respect some further remarks are in order here.

Firstly, the chaotic possibility emerging from dynamic logit models evokes the need to raise the question whether other types of discrete choice models arising from random utility theory (such as probit, nested logit, etc.) can exhibit, under certain conditions, chaotic behaviour; in other words, whether chaos can appear not only for actors whose behaviour can be represented by multinomial logit models suffering from the well-known IIA assumption.

Secondly, since chaos appears in logit models for high values of the marginal utility α_j 's we have to examine the economic or physical conditions producing α_j and hence impossible predictions on the actor's behaviour. In this context the measurement of the values of the parameters and their estimation appear necessary for checking the consistency of the models with chaos.

Acknowledgements

The second author would like to thank the computer assistance by Andrea Gerali and Luca Schionato (University of Bergamo). Also the grant C.N.R. n. 89.01069.11 is gratefully acknowledged.

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