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**AN EQUIVALENCE OF COMMUNICATION PROTOCOLS FOR
INTERCONNECTION NETWORKS**

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AN EQUIVALENCE OF COMMUNICATION PROTOCOLS FOR INTERCONNECTION NETWORKS

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Abstract. The stop and retransmission communication protocol are shown to be equivalent for a class of product form interconnection networks. Applications include but are not limited to architectures of

- . Interconnection Metropolitan Area Networks, and
- . CSMA-protocols such as BTMA.

1. Introduction

Since the introduction of the ALOHA-communication system in the early seventies, a large variety of (tele-)communication protocols has been investigated of which most notably Carrier Sense Multiple Access Schemes (CSMA) (e.g. [4], [18], [19], [20]). Under exponentiality assumptions a number of these have been shown to exhibit a closed product form expression, such as the Busy Tone Multiple Access Scheme (BTMA) and the Rude-CSMA protocol, which take into account the hidden terminal collision problem (cf. [1], [5]).

Recently, (cf. [2], [3], [4], [21]) these results were unified and generalized to more complex random access protocols as well as general transmission and scheduling times. Relatedly, exact product form solutions have been established for interconnected Metropolitan Area Networks (cf. [16]).

These product form results, however, were all established under the assumption of blocked messages to be lost or equivalently to be retransmitted. In practice, however, the "stop" communication protocol is often in order under which the scheduling of a next transmission is delayed or stopped if it would lead to blocking.

This note will show that the "stop" and "retransmitting" protocol are effectively the same for a wide class of non-exponential product form interconnection networks. This equivalence is intuitively clear for the exponential case but far from obvious when non-exponential transmission and scheduling times are involved, and cannot be concluded directly from literature (see remarks 2.2 and 2.3). As will be illustrated, the class of interconnection networks to which the result applies includes for instance

- . CSMA/BTMA-schemes.
- . A circuit switching network.
- . A metropolitan area network.

2. General model and equivalence result.

Consider a system of a fixed number of N sources (e.g. transmitters). Each source is alternatively in an "idle" and "busy" mode during which it requires an idle service (e.g. the scheduling of a transmission) and

a busy service (e.g. a transmission of a message) respectively. With state $H = \{h_1, \dots, h_n\}$ denoting that n sources are currently busy, being sources $h_1, \dots, h_n$, the possible states, however, are restricted to

$$H = \{h_1, \dots, h_n\} \in C$$

where C is some set of states such that

$$\{h_1, \dots, h_n\} \in C \Rightarrow \{h_1, \dots, h_{j-1}, h_{j+1}, \dots, h_n\} \in C \quad (j=1, \dots, n).$$

To meet this condition one of the following two protocols is in order.

Stop protocol (P_1). As soon as the system is in state H , the servicing (e.g. the scheduling of a transmission) of any idle source h such that $H+h$ is not contained in C (i.e. $H+h \notin C$) is blocked and interrupted. As soon as the state changes in a state H' where $H'+h \in C$ this service becomes unblocked and its servicing is resumed to complete the residual service requirement (e.g. time to complete a scheduling). When its idle service is completed, a source becomes busy and starts a busy service (e.g. a message transmission) after which it becomes idle again.

Retransmission Protocol (P_2). The servicing of idle services is never interrupted. In contrast, when the system is in state H , an idle source h which completes its idle service has to undergo a total new idle service (e.g. to reschedule a transmission) if $H+h \notin C$. When $H+h \in C$ it becomes busy and starts its busy service (e.g. transmission) after which it becomes idle again.

Remarks 2.1.

(1) In the exponential case it seems intuitively clear that both protocols are effectively the same as a residual exponential service and a total exponential service are stochastically equal. In the non-exponential case this is no longer obvious and in fact an equivalence result will not generally hold. Based on a product form, or rather partial balance results, however, it will be proven to be true in the present setting.

(2) The retransmission protocol has been assumed in all the references mentioned in the introduction that provided product form results for communication networks. It also corresponds to the so-called "triggering" or "recirculating" blocking protocol that has been commonly used in the literature on queueing networks with blocking (e.g. [7], [11], [13]).

(3) A restricted set of states C of the form (2.1) can be called "coordinate convex" and directly corresponds to similar multi-class constraints in the literature on queueing (cf. [7], [11], [13]). In the next section some typical present-day communication applications will be given.

Throughout, let a state $H = \{h_1, \dots, h_n\}$ denote that components $h_1, \dots, h_n$ are busy (say in increasing order), and denote by H^+h the state in which component h is added to (+ sign) or deleted from (— sign) H as a busy source. Furthermore, without loss of generality assume that the random idle and busy services are all independent with for source h distribution functions $A_h(\cdot)$ and $B_h(\cdot)$, density functions $a_h(\cdot)$ and $b_h(\cdot)$, and means α_h and β_h respectively. Let the state

$$[S, T] = ((s_1, t_1), \dots, (s_N, t_N))$$

denote that source i is in mode s_i , where $s_i=1$ stands for idle and $s_i=2$ for busy, with a residual time t_i up to completion of the current idle when $s_i=1$, or busy service when $s_i=2$. For a given specification $S = (s_1, \dots, s_N)$, let H denote the corresponding busy sources and write $\pi_i([S, T])$ and $\pi_i(H)$ for the stationary density and stationary probability of states (S, T) and H under protocol P_i , $i=1, 2$. The following two theorems will be proven, which show that both protocols lead to exactly the same product form. The first is the key theorem. The second is the more practical form.

Theorem 1. *With c a normalizing constant, we have*

$$\pi_1([S, T]) = \pi_2([S, T]) = c \prod_{\{h: s_h=1\}} [1-A_h(t_h)] \prod_{\{h: s_h=2\}} [1-B_h(t_h)] \quad (1)$$

Proof. We need to verify the global balance or forward Kolmogorov equations under either protocol where without loss of generality we assume that these have a unique solution. To this end, for a given state (S, T) and source i , denote by

$$[S, T] = (s_i, t_i)_i + (\hat{s}_i, \hat{t}_i)_i$$

the same state with the specification for source i changed from (s_i, t_i) in $(\hat{s}_i, \hat{t}_i)$. Further, we write 0^+ for a right hand limit at 0 and $1(A)$ for the indicator of an event A , i.e. $1(A)=1$ if A is satisfied and 0 otherwise. The global balance equations can be derived in a standard manner by considering a point of time t , conditioning upon time $t-\Delta t$, dividing by Δt , and letting $\Delta t \rightarrow 0$. Then, for a fixed state $[S, T]$ with busy sources represented by $H \in C$, the global balance equations become:

P_1 (Stop protocol)

$$\begin{aligned} \sum_{\{h: s_h=1\}} \left\{ \frac{\partial}{\partial t_h} \pi_1([S, T]) 1(H+h \in C) \right. \\ \left. + \pi_1([S, T] - (1, t_h)_h + (2, 0^+)_h) a_h(t_h) 1(H+h \in C) \right\} + \\ \sum_{\{h: s_h=2\}} \left\{ \frac{\partial}{\partial t_h} \pi_1([S, T]) + \pi_1([S, T] - (2, t_h)_h + (1, 0^+)_h) b_h(t_h) \right\} = 0 \end{aligned} \quad (2)$$

P_2 (Retransmission protocol)

$$\begin{aligned} \sum_{\{h: s_h=1\}} \left\{ \frac{\partial}{\partial t_h} \pi_2([S, T]) \right. \\ \left. + \pi_2([S, T] - (1, t_h)_h + (1, 0^+)_h) a_h(t_h) 1(H+h \notin C) \right. \\ \left. + \pi_2([S, T] - (1, t_h)_h + (2, 0^+)_h) a_h(t_h) 1(H+h \in C) \right\} + \\ \sum_{\{h: s_h=2\}} \left\{ \frac{\partial}{\partial t_h} \pi_2([S, T]) + \pi_2([S, T] - (2, t_h)_h + (1, 0^+)_h) b_h(t_h) \right\} = 0. \end{aligned} \quad (3)$$

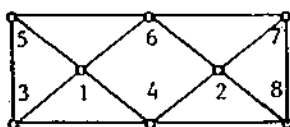
Finally, truncation results as from [12], section 1.6, or [22], definition 3.7.2, for exponential reversible networks do not apply, as the non-exponential case considered herein essentially comes down to a non-reversible structure such as illustrated above for Erlang distributions.

3 Applications

In this section we provide some direct applications of present-day interest for which the equivalent insensitive product form result (4) holds. Under the stop protocol they all seem to be new.

3.1 CSMA/BTMA-protocol (cf. [1], [2], [3], [4], [18], [19], [20], [21])

(i) *CSMA* Let the sources correspond to transmitters that can be graphically represented such that adjacent sources (neighbors) cannot be busy (transmit) at the same time. In practice this is achieved by the so-called "Carrier Sense Multiple Access" (CSMA)-scheme in which a transmitter senses the state of its channels just prior to starting a transmission and where upon sensing a busy channel from a neighbor the transmission is aborted (inhibited). For example, in the figure below a transmission from source 1 prohibits any source 3, 6 to start a transmission.



With $N(h)$ the set of all neighbors of source h , condition (1) is guaranteed by

$$C = \{H \mid h_2 \notin N(h_1) \text{ for all } h_1, h_2 \in H\} \quad (5)$$

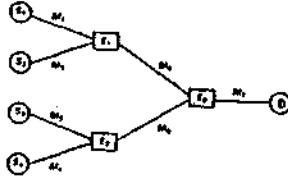
(ii) *BTMA* In the example above, sources 1 and 2, for example, which are outside hearing range can transmit at the same time. This will lead to a collision at nodes 3 and 4 which in turn will result in lost messages. This is known as the "hidden terminal problem". To eliminate this problem, the so-called Busy Tone Multiple Access (BTMA)-scheme has been introduced (cf. [18]). Under BTMA a node which senses a busy channel (in other words, which hears a transmitting neighbor) broadcasts a busy tone to all its neighbors to prevent idle neighbors from starting a transmission.

The set C from (5) now still applies (i.e., satisfies (1)), provided we replace $N(h)$ by the set of all one and two-link neighbors (e.g. $N(5) = \{2, \dots, 7\}$).

Remark The insensitive product form result (2) for both the CSMA and BTMA protocol has been reported (cf. [1], [2], [3], [4], [14], [21]) under the retransmission (or loss) protocol. As yet, for the "stop" protocol no such result seems to be available.

3.2 Hierarchical circuit switching (cf [5], [21])

Consider a circuit switching network with 4 different types of sources with a fixed path along which a message from that source to the destination is to be transmitted. This transmission requires one trunk from each trunkgroup along this path. Interference thus arises with limited trunkgroups and messages using the same trunkgroups.



With M_i the number of trunks in trunkgroup i and n_i the number of busy sources of type i , condition (1) is satisfied by C the set of states H such that:

$$n_i \leq M_i \quad (i=1, \dots, 4)$$

$$n_1 + n_2 \leq M_5$$

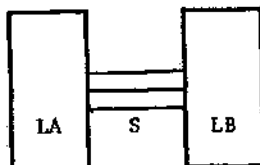
$$n_3 + n_4 \leq M_6$$

$$n_1 + n_2 + n_3 + n_4 \leq M_7.$$

Remark The insensitive product form result (2) for this example has been proven by the results in [5], [7], [11] and [21] under the retransmission (or lost and triggering) protocol. For the stop protocol it seems to be new.

3.3 Interconnected Metropolitan Area Networks (MAN's) (cf. [16])

Consider a communication system with two groups of subscribers, say a group A and B with M and N subscribers, such as representing two metropolitan or local area networks. Both within a group and in between the groups communication between subscribers might be possible. To this end, number all subscribers $1, \dots, M+N$ and identify each possible connection from a source subscriber m to a destination subscriber n as a source (m,n) . The description of section 2 now applies by saying that a connection is busy when a transmission along this connection takes place and idle otherwise and assuming some circuit allocation policy which restricts the feasible busy configuration, to some "coordinate convex" region C . We give some examples below.



Example 1 (Limited total number of circuits) (cf. [16]). For a given state H of busy connections let n_A , n_B and $n_{A,B}$ denote the number of busy connections within A, within B and in between A and B respectively. Assume finite numbers of LA and LB local circuits within A and B and S circuits in between A and B. Then the model of [16] is included by

$$C = \{H \mid n_A \leq LA, n_B \leq LB, n_{A,B} \leq S\}$$

for the "dedicated allocation policy" with separate circuits for local and long-distance transmissions and by

$$C = \{H \mid n_A \leq LA+S, n_B \leq LB+S, 0 \leq n_{A,B} \leq S-(n_A-LA)^+-(n_B-LB)^+\},$$

where $(y)^+=0$ for $y \leq 0$ and y^+ for $y > 0$, for the "shared allocation policy" in which the inter-MAN circuits are shared among local and long-distance calls. As another shared allocation policy, each long-distance connection may require a local circuit within each local area, which is reflected by

$$C = \{H \mid n_A+n_{A,B} \leq LA, n_B+n_{A,B} \leq LB, n_{A,B} \leq S\}$$

Example 2 (Excluding connections). Certain connections may have to be excluded to be busy at the same time. For example, exclusion of busy connections (m,n) and (n,m) at the same time reflects one-way communication systems such as in air traffic. The corresponding set of admissible states is "coordinate convex" by:

$$1((m,n) \in H) + 1((n,m) \in H) \leq 1 \quad (\forall (n,m)).$$

Remark The results in [16] assume exponential idle and busy services and the blocking-calls-cleared (=retransmission) protocol.

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